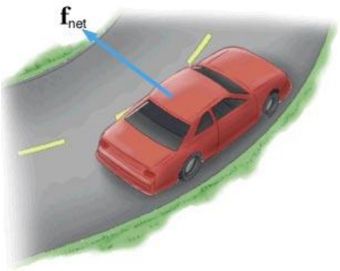
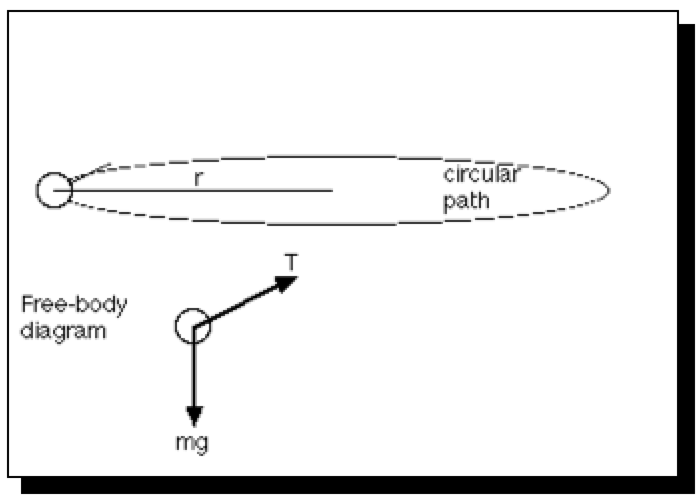
Physics 3204
Unit 1:
Section 3: Uniform Circular Motion (UCM)

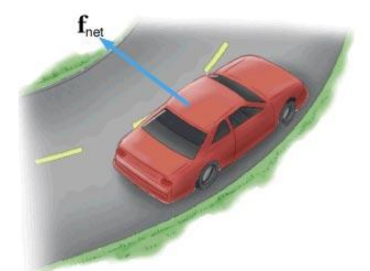


Uniform Circular Motion (UCM)

- **Uniform Circular Motion (UCM)** is defined as motion in circle at a constant speed - **not velocity** since the direction is constantly changing, thus the object is accelerating.

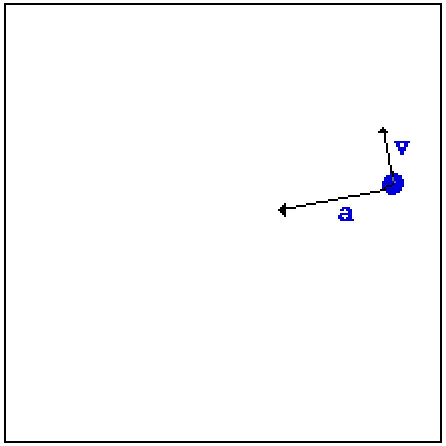
Example: Mass tied to a string and whirling in a circle





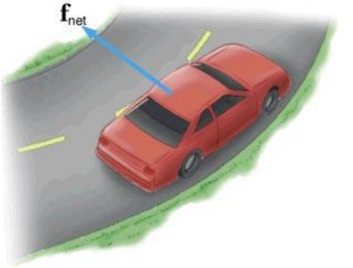
This acceleration is called **centripetal acceleration** and is always directed towards the center of a circle.

Centripetal - Circle Seeking

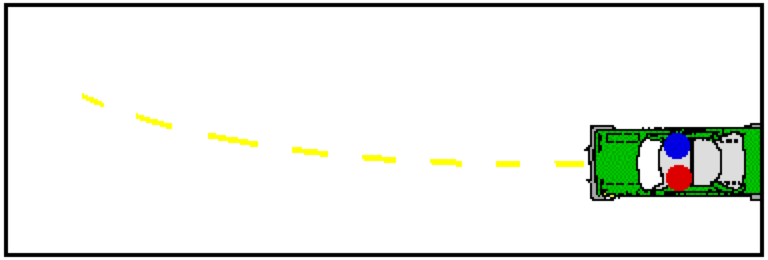


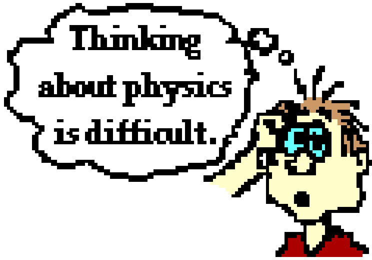
An object undergoing uniform circular motion is moving with constant speed; nonetheless, it is accelerating due to its change in direction. The direction of the acceleration is inwards

Note that velocity is tangential to the circle

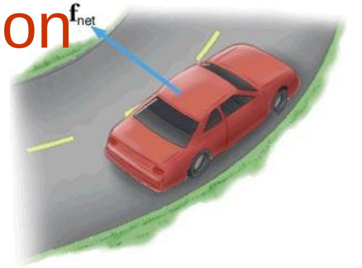


Many students of physics do not believe in centripetal ("inwards") forces. Perhaps the reason for adhering to this misconception stems from their experiences with riding as a passenger in automobiles and amusement park rides.





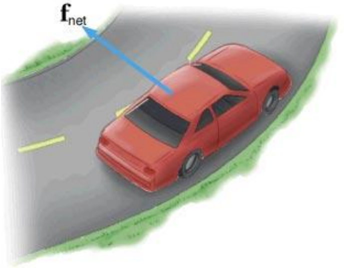
An inward net force is required to make a turn in a circle. This inward net force requirement is known as a centripetal force requirement. In the absence of any net force, an object in motion (such as the passenger) continues in motion in a straight line at constant speed (Newton's 1St Law). The inward net force directed perpendicular to the velocity vector, the object is always changing its direction and undergoing an inward acceleration.



Centripetal acceleration (a_c) is found using the following formula:

$$a_c = \frac{v^2}{r}$$

a_c=> Centripetal Acceleration (m/s^2)
v=> speed (m/s)
r => radius (m)



For a circle

The speed of an object undergoing U.C.M. is found by

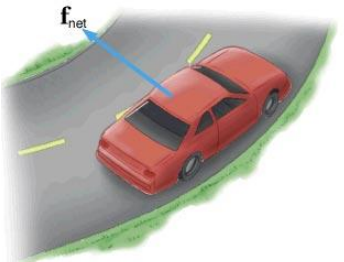
$$v = \frac{d}{t} \, or \, \frac{circumference}{Period} = \frac{2\pi r}{T} = 2\pi r f$$

Recall that

$$f = \frac{1}{T}$$

Frequency (f) => number of cycles per sec ($\frac{1}{T}$). Unit of measure is Hertz (Hz)

Period (T): time for one cycle. Unit of measure is seconds



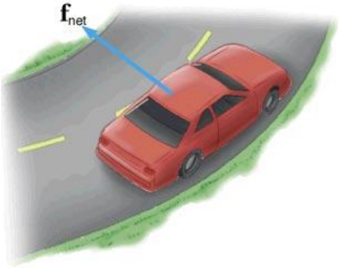
If an object is accelerating towards the center, there must be a force towards the center. This center -seeking for is called the centripetal force- defined as a force that causes an object to move in a circle. It can be supplied through a mass on a string; a frictional force (car rounding a curve) or a gravitational force (moon around the earth)

U.C.M is really a special case of Newton's Second Law

$$F_{net} = ma$$

$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r}$$



Also note that $v = \frac{2\pi r}{T}$ thus

$$F_c = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r}$$

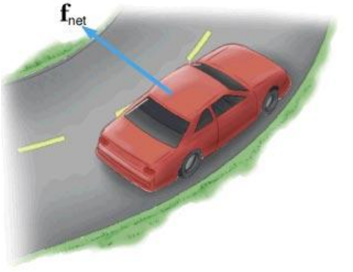
$$= \frac{m4\pi^2 r^2}{rT^2}$$

$$F_c = \frac{4\pi^2 rm}{T^2}$$

or

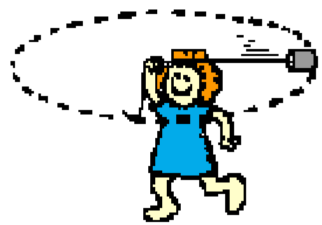
$$F_c = 4\pi^2 rmf^2$$

Acceleration is uniform in U.C.M. thus the force that causes it must also be uniform

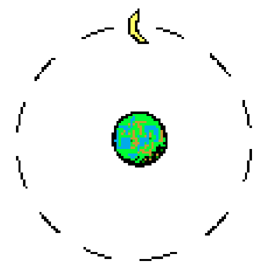


Any object moving in a circle (or along a circular path) experiences a centripetal force; that is there must be some physical force pushing or pulling the object towards the center of the circle.

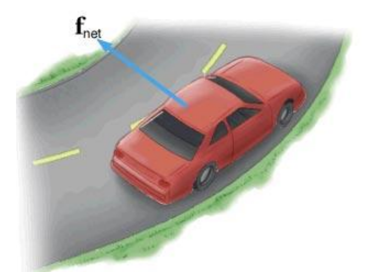
As a car makes a turn, the force of friction acting upon the turned wheels of the car provide the centripetal force required for circular motion.



As a bucket of water is tied to a string and spun in a circle, the force of tension acting upon the bucket provides the centripetal force required for circular motion.



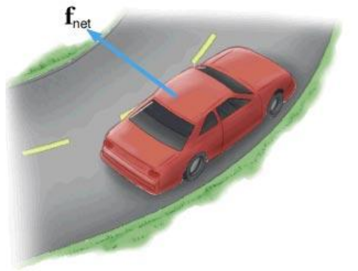
As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.



Example 1:

A 150g ball at the end of a string is revolved uniformly in a horizontal circle of radius 0.500 m. The ball makes exactly 2.00 revolution in a second.

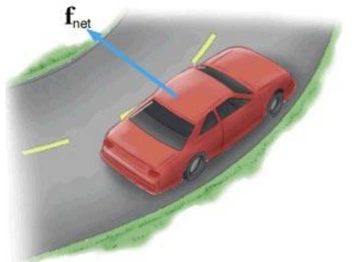
A) Find the speed of the Ball
B) Finds it Centripetal Acceleration



Givens:

m = 150g = .15 kg
r=.500 m
RPS= 2.00

$$T = \frac{time}{\#cycles} = \frac{1rev}{2.00rev/s} = ..50s$$



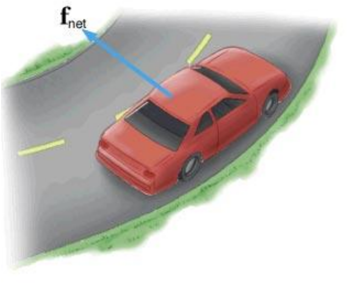
A)

$$v = \frac{2\pi r}{T}$$

$$= \frac{2(3.14)(.500}{.5}$$

$$= 6.3 m/s$$

Note v decreases with increase in period



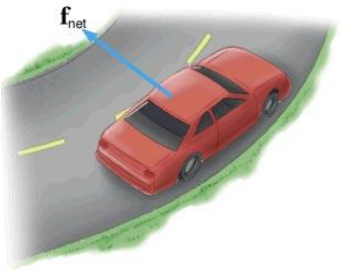
B)

$$a_c = \frac{v^2}{r}$$

$$= \frac{(6.3)^2}{0.500m}$$

$$= 79m/s^2$$

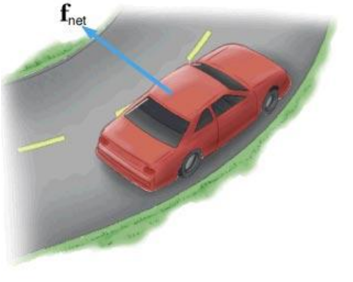
Toward the Center



Example 2

A ball of 200 g moves at a constant speed, in a circle that is parallel to the ground. Find the tension (F_t) in string (length 0.08m) for speeds of

A) 5.0m/s
B) 10 m/s



A)

$F_c = ma_c$

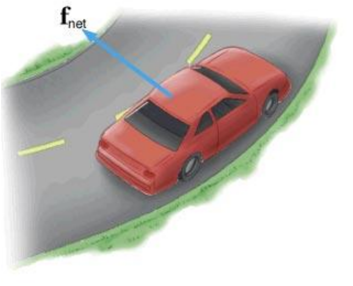
$$F_t = \frac{mv^2}{r}$$

$$= \frac{(0.200)(5.0m/s)^2}{080m}$$

$$= 6.25N$$

$0r$

$$= 6.3N$$



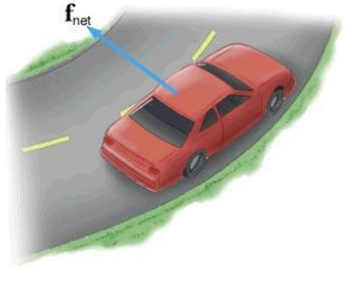
B)

$$F_t = \frac{mv^2}{r}$$

$$= \frac{(0.200)(10.0m/s)^2}{080m}$$

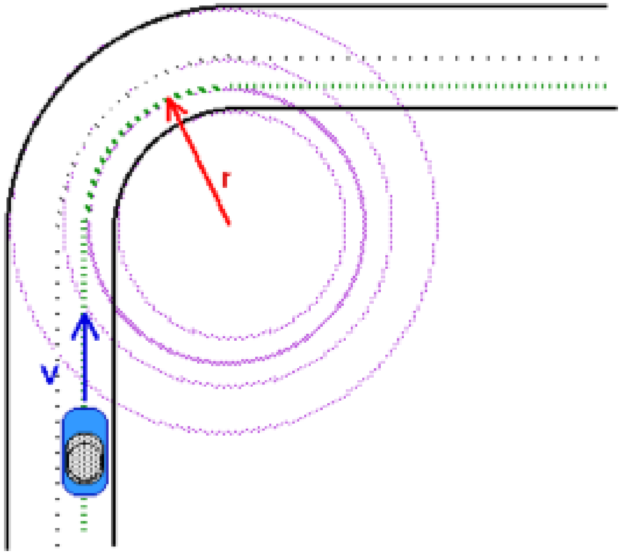
$$10N$$

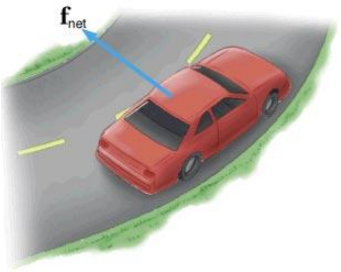
There should be greater tension with increase in speed



Example 3:

A car goes around a flat, circular track of radius 200 m at a constant speed of 90 km/h. What is the acceleration of the Car?





Given

r=200m

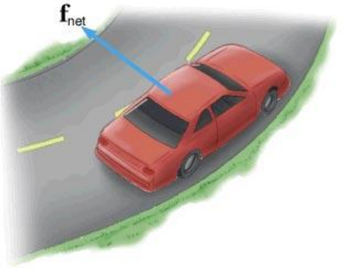
v=90 km/hr= 25 m/s

$$a_c = \frac{v^2}{r}$$

$$= \frac{(25m/s)^2}{200m}$$

$$= 3.1m/s^2$$

3.1 m/s^2 towards the center

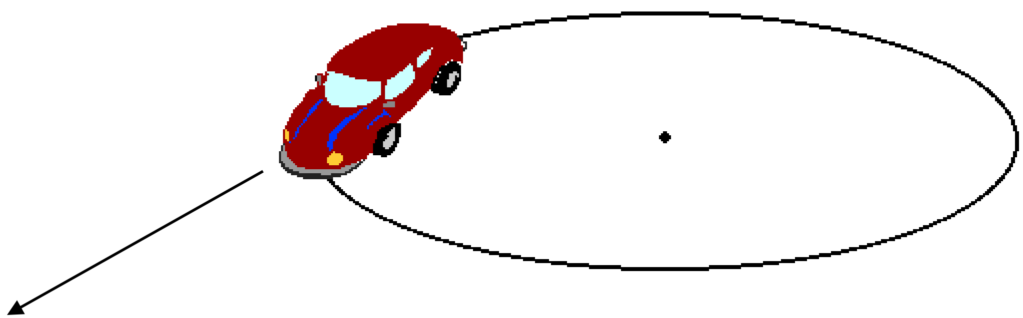


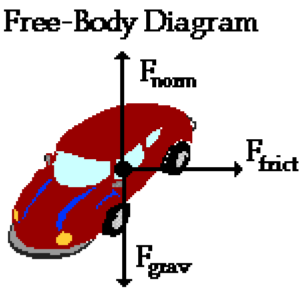
Question:
There is no string attached to the car. Therefore, what forces causes the car to move in a circle?

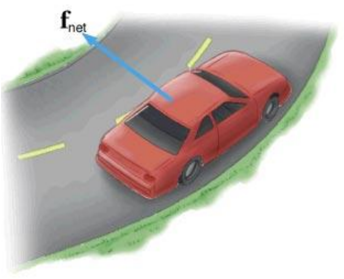
Answer:

The force of static friction between the tires and the road.

FBD







Example 4:

If the mass of the car in example 3 is 1000 kg.

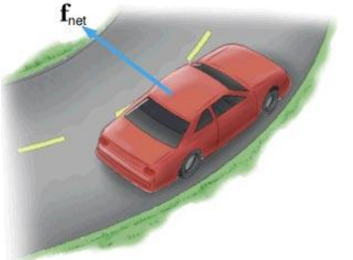
A) What is the frictional force required to provide the acceleration?

$F_{net}=F_c=F_s=ma_c$

$=(1000kg)(3.1m/s^2)$

= (3100 N [towards the center]

Note: F_c is the net force to keep the object on the track

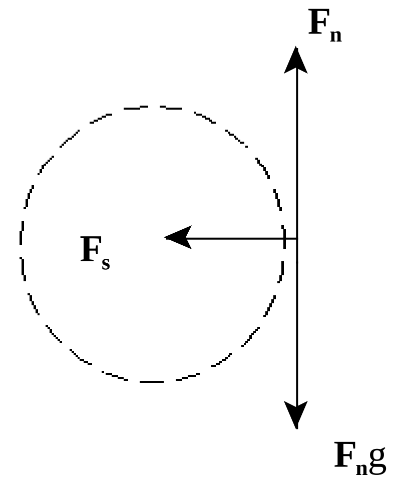


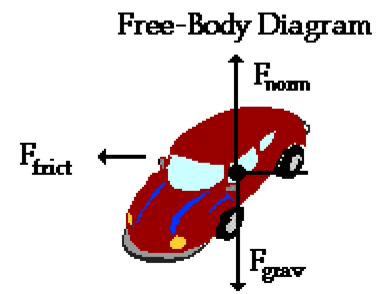
B) If the coefficient of static friction is 0.80, what is the maximum speed at which the car can circle the track

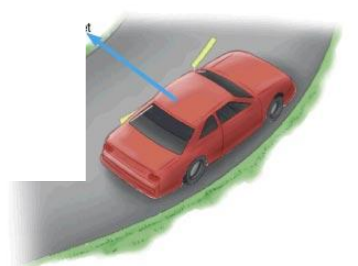
<u>Y direction</u>

a=0 thus

$F_n = F_g = mg$







$F_s = \mu_s F_n$

$F_s = F_c$

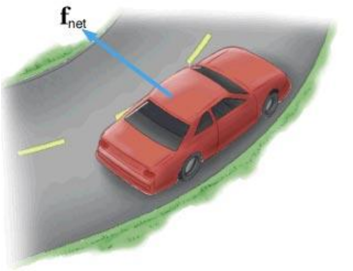
$$\mu_s mg = \frac{mv^2}{r}$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{(0.80)(9.8)(200)}$$

$$= 40 m / s$$

From example 4(b) we can determine that mass of the car has no effect on the speed of the car on the track

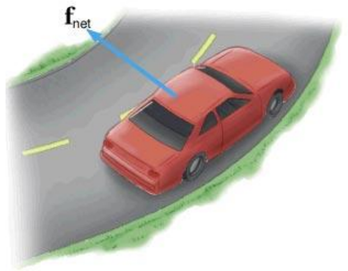


Example 5 (You try)

A 1200 kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/hr. Will the car make the turn or will it skid if:

A) The pavement is dry and the coefficient of static friction is 0.60.

B) The pavement is icy and $\mu_s = 0.25$



A)

$F_s = F_c$

$$\mu_s mg = \frac{mv^2}{r}$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{(0.60)(9.80)(50)}$$
$$= 17 m/s$$
$$= 62 km/hr$$

Since the max. Speed is 62 km/hr the car will <u>not</u> skid

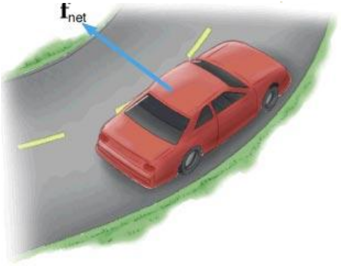
B)

$$\mu_s mg = \frac{mv^2}{r}$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{(0.25)(9.80)(50)}$$
$$= 11 m/s$$
$$= 40 km/hr$$

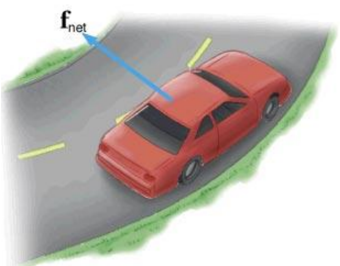
Since the max. Speed is 40 km/hr the car will skid and not make the turn



Activity

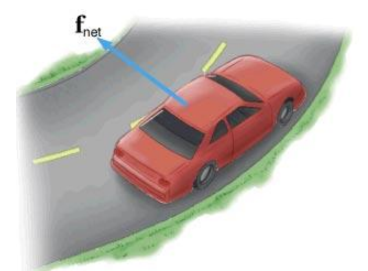
Questions:

- Do 1-6 page 206

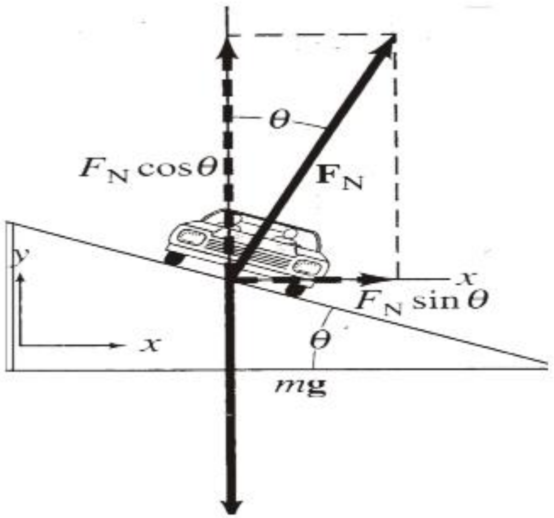


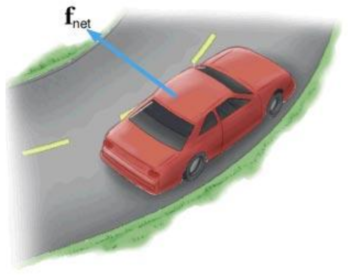
Banked Curves (page 210-211)

- Good highways have banked or slanted curves so that the normal force exerted by the road on the car has a horizontal component . This horizontal component can provide part of or all the force need to produce centripetal force instead of relying on the force of static friction. Consequently the is much safer, especially under slippery conditions.



Free Body Diagram of a Car rounding a banked curve





Y-Dir (no movement)

$F_{net}=ma=0$

$$F_N \cos\theta - mg = 0$$

$$F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta}$$

X-Dir

$F_{net}=ma$

$$F_N \sin\theta = \frac{mv^2}{r}$$

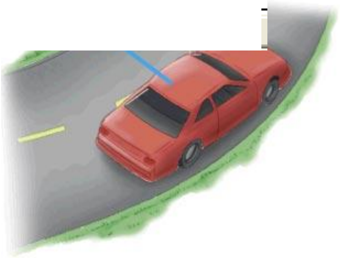
Substitute F_N from y-dir

$$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$$

$$g\tan\theta = \frac{mv^2}{r}$$

$$v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$



To find the speed of object around a bank curve, we use the following

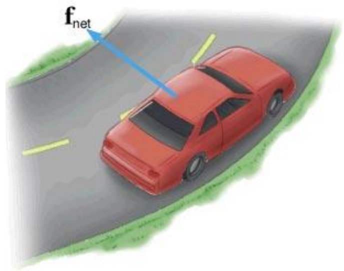
$$v^2 = rg\tan\theta$$

or

$$v = \sqrt{rg\tan\theta}$$

Where:

r= radius , g= gravity, and θ is the banking angle



Example 1:

A curve of radius 900 m is banked. If f=0 and the object is moving around it at 30 m/s without slipping, what is the banked angle?

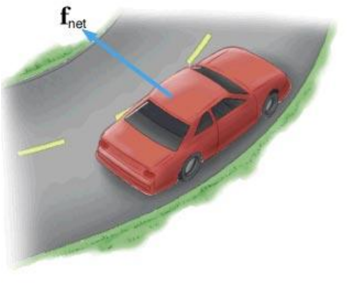
$$v^2 = rg\tan\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\tan\theta = \frac{(30m/s)^2}{(900m)(9.8m/s^2)}$$

$$\theta = \tan^{-1}(0.102)$$

$$\theta = 6^0$$



Example 2:

A race car travels along a banked curve at a speed of 120 km/hr. It doesn't depend on the force of friction to keep it on the track. If the turn is banked at an angle of 250 to the horizontal, what is the radius of rotation?

Given:

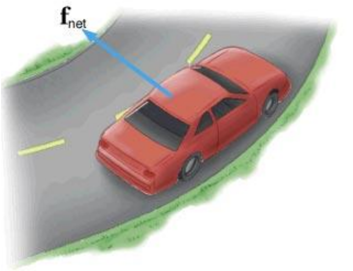
v = 120 km/h = 33.3 m/s
Tan θ= 25°

$$v^2 = rg\tan\theta$$

$$r = \frac{v^2}{g\tan\theta}$$

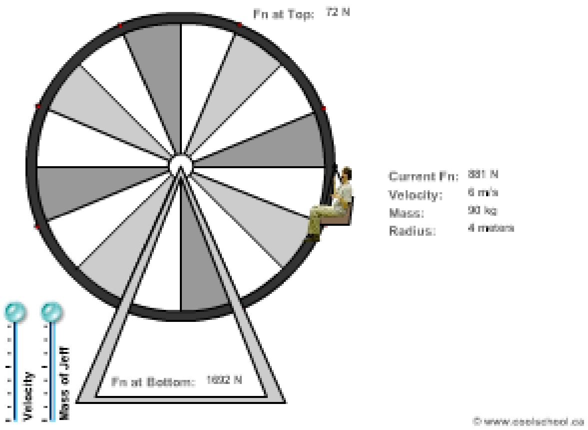
$$= \frac{(33.3m/s)^2}{(9.8m/s^2)(\tan 25^0)}$$

$$= 243m$$

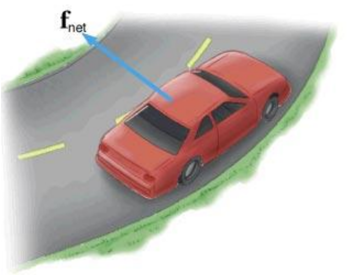


VERTICAL CIRCULAR MOTION (P.208)

- **Actually "non-uniform circular motion" since the speed changes as you move vertically around a circle.**
- **There are two important point of interest- the top and bottom of the circle. To find the speed or other info. at these points it is important to draw FBD's**

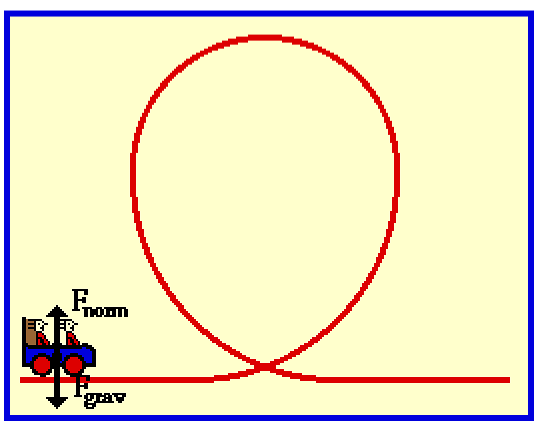


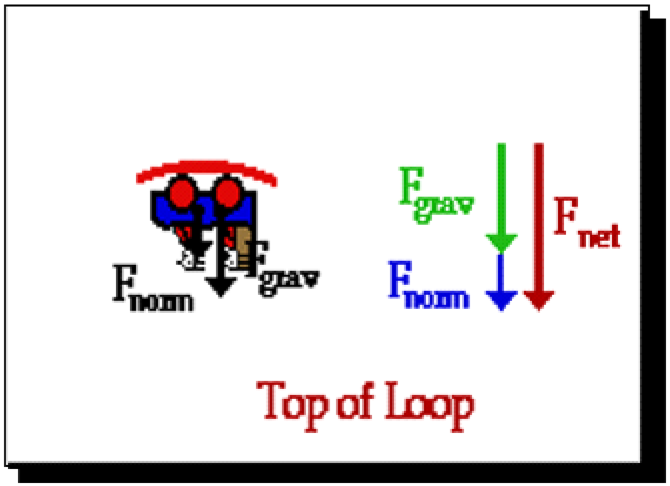
Note: Direction System - let positive be towards centre of the circle

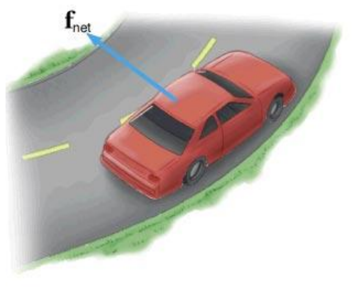


Example 1:

A roller-coaster car enters the circular- loop portion of the ride. At the very top of the circle (where the people in the car are upside down), the speed of the car is 25 m/s and the acceleration is straight down. If the diameter of the loop is 50m and the total mass of the car and passengers is 1200 kg, Find the magnitude and direction for the formal force exerted by the track on the car at this point.







Note: Direction System - let positive be towards centre of the circle

$$F_{net} = ma$$

$$F_c = ma_c$$

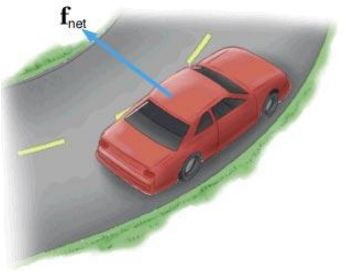
$$Fg + Fn = \frac{mv^2}{r}$$

$$F_n = \frac{mv^2}{r} - F_g$$

$$= \frac{(1200kg)(25m/s^2)^2}{(25m)} - (1200)(9.80N/Kg)$$

$$= 30{,}000N - 11{,}760N$$

$$= 18{,}240N \text{ or } 1.8x10^4\ N[down]$$



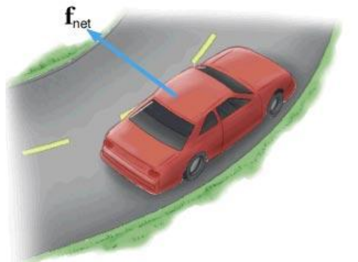
Example 2:

Question

In question 1: if the net force on the car at its top -must point is straight down then why doesn’t the car fall straight down?

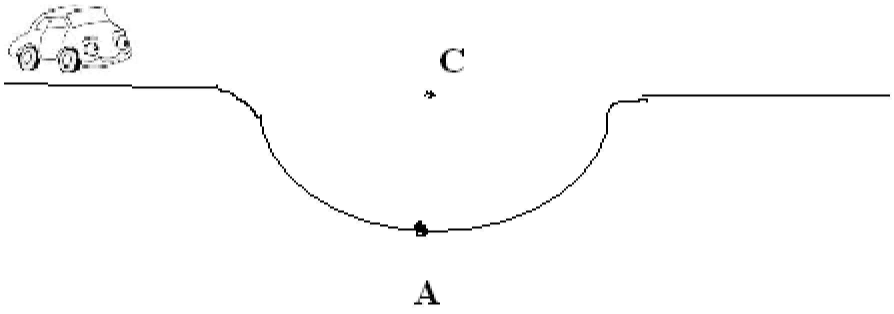
Answer:

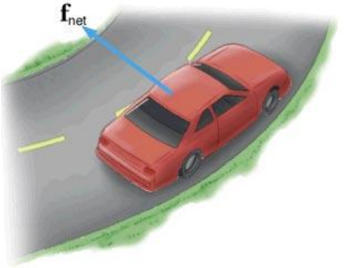
If the car had zero velocity at this point it would fall down but it does not (25 m/s). The velocity is tangent to the circle and carries the car forward, the car will keep it moving in a circle.



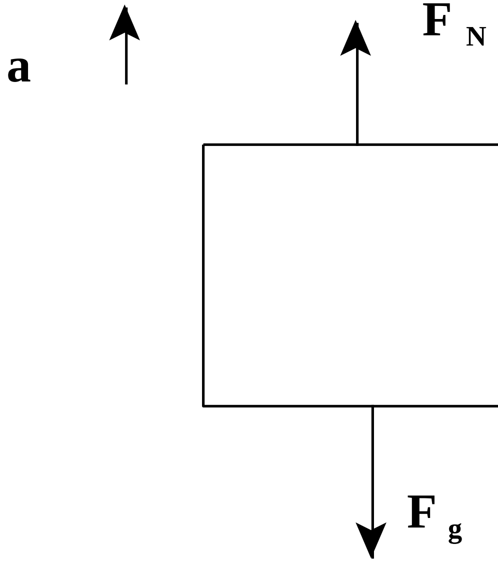
Example 3:

A 1200 kg car is being driven at a constant speed of 20 m/s, along a country road as shown. Determine the force on the car by the road at point A if the radius of curvature of the dip in the road us 60m.

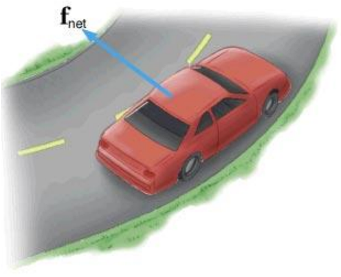




FBD at point A



Remember: towards center is positive



$$F_{net} = ma$$

$$F_c = ma_c$$

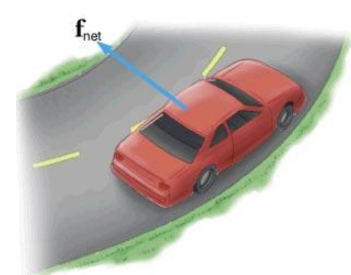
$$F_n - F_g = \frac{mv^2}{r}$$

$$F_n = \frac{mv^2}{r} + F_g$$

$$= \frac{(1200kg)(20m/s)^2}{(60m)} - (1200)(9.80N/Kg)$$

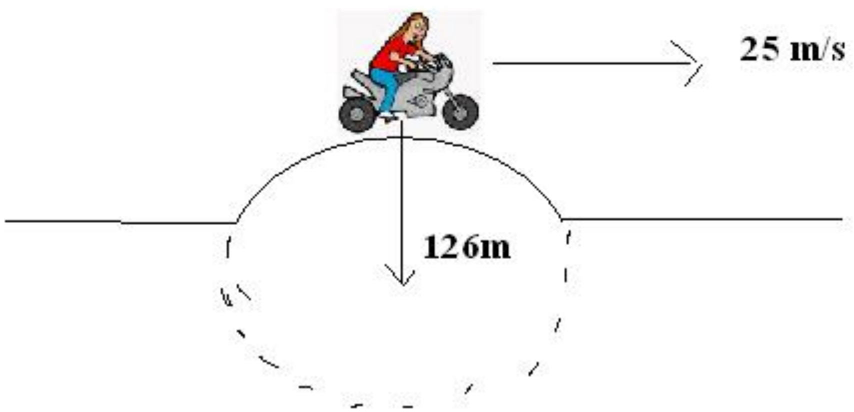
$$= 8000N + 11{,}760N$$

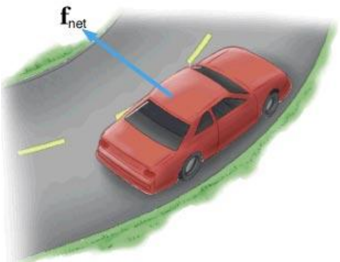
$$= 19760N \; or \; 12.0x10^4 \; N[up]$$



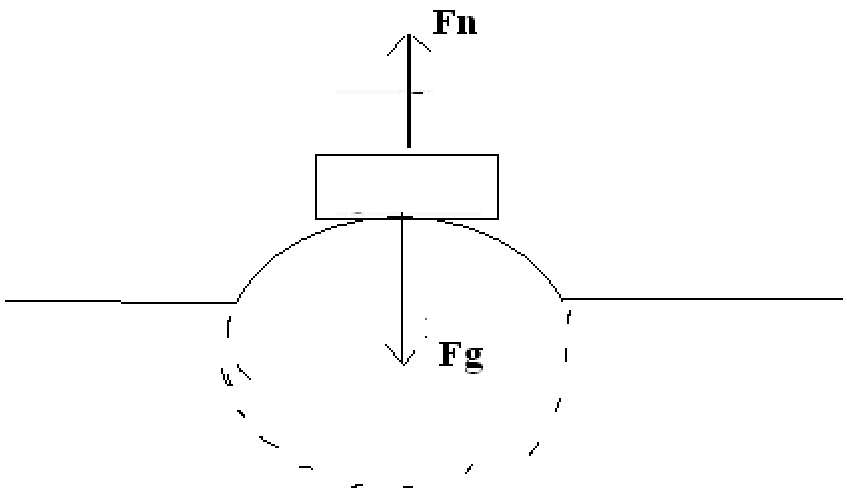
Example 4:

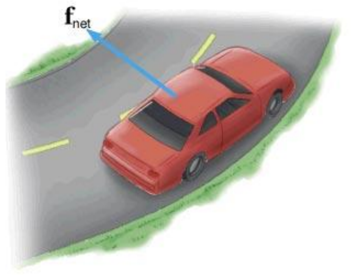
A motor cycle has a constant speed of 25 m/s as it passes over the top of a hill whose radius of curvature (in the vertical plane) is 126 m. The mass of the driver and the cycle is 342 kg. Find the magnitude and the direction of the normal force that acts on the motor cycle?





FBD:





$$F_c = ma_c$$

$$F_g - F_n = \frac{mv^2}{r}$$

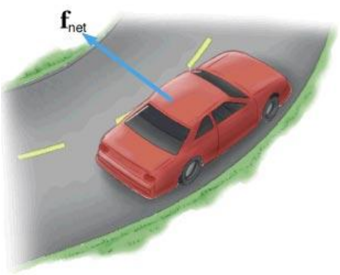
$$= (342kg)(9.80N / Kg) - \frac{(342kg)(25m / s)^2}{(126m)} -$$

$$-F_n = \frac{mv^2}{r} - F_g$$

$$= 3351.6N - 1696.4N$$

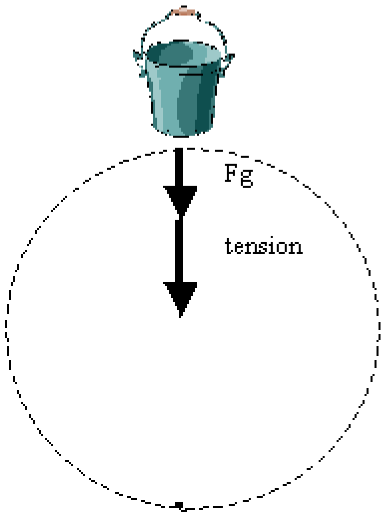
$$= 1655N \; or \; 1.7x10^3 N[up]$$

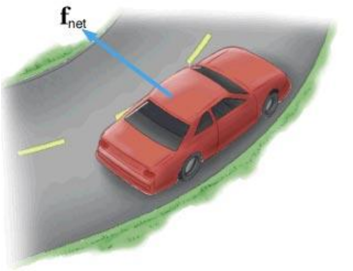
$$F_n = F_g - \frac{mv^2}{r}$$



Example 5

A student whirls a bucket of water in a vertical circle fast enough so that the water does not fall out. If the student’s arm is 0.75 m long, what is the minimum speed at which the bucket must be swung?





$$F = ma$$

$$F_c = ma_c$$

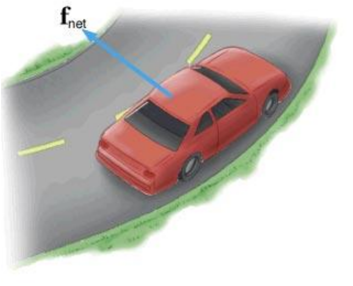
At the top of the circle the centripetal forces are due to

$F_{gravity}$ and $F_{tension}$:

$F_{gravity} + F_{tension} = ma_c$

Since we are trying to determine the minimum velocity to keep the bucket moving, look at the point where $F_{tension}$ becomes 0 N.

$F_{gravity} + 0 = ma_c$



$$mg = \frac{mv^2}{r}$$

$$g = \frac{v^2}{r}$$

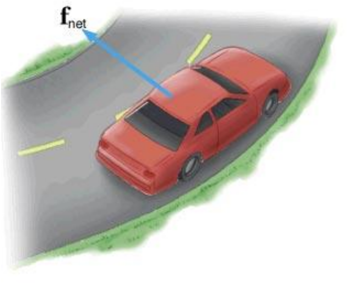
$$v^2 = rg$$

$$v = \sqrt{rg}$$

$$v = \sqrt{(0.75m)(9.80\frac{m}{s^2})}$$

$$v = 2.7\frac{m}{s}$$

Thus the bucket must be swung at a minimum speed of 2.7 m/s.

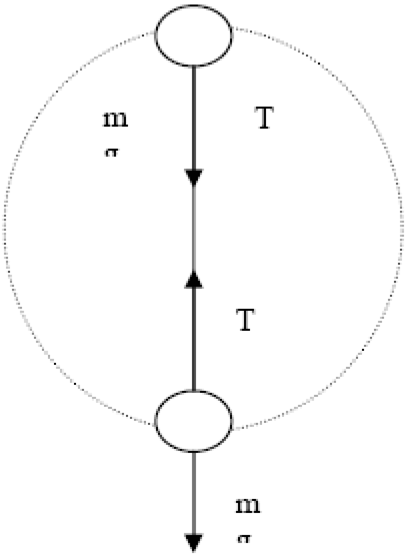


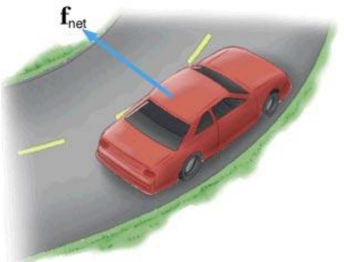
Example 6

A ball on the end of a string is revolving at a uniform rate in a vertical circle of radius 96.5 cm as shown. If its speed is 3.15 m/s and its mass is 0.335 kg, what is the tension in the string when the ball is,

a) at the top of its path?

b) at the bottom of its path?





This solution assumes that towards the center of the circle is a positive direction.

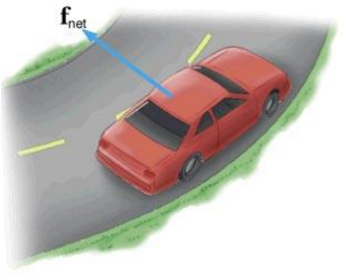
a) $\vec{T} + \vec{F}_g = \vec{F}_c$

$$T = F_c - F_g$$

$$T = \frac{mv^2}{r} - mg$$

$$T = \frac{(0.335kg)(3.15\frac{m}{s^2})^2}{0.965m} - (0.335kg)(9.80\frac{m}{s^2})$$

$$T = 0.162N$$



$$\vec{T} + \vec{F}_g = \vec{F}_c$$

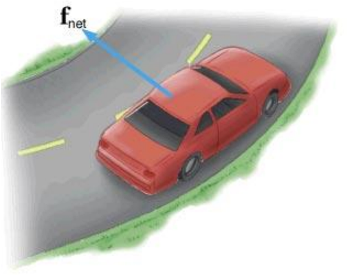
$$T - F_g = F_c$$

$$T = F_c + F_g$$

$$T = \frac{mv^2}{r} + mg$$

$$T = \frac{(0.335kg)(3.15\frac{m}{s^2})^2}{0.965m} + (0.335kg)(9.80\frac{m}{s^2})$$

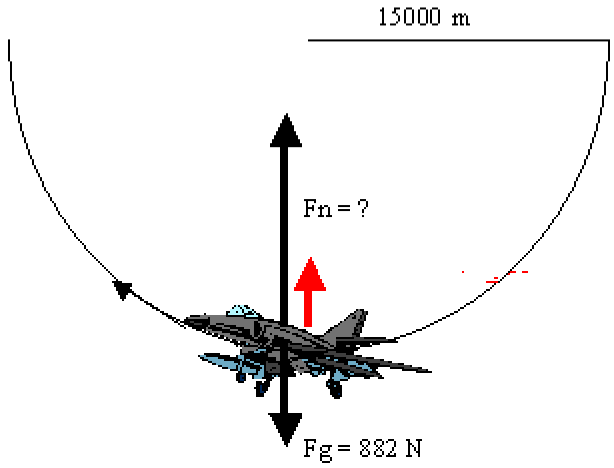
$$T = 6.73N$$



Example 7

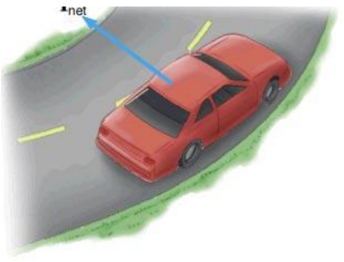
A fighter jet completes a maneuver causing it to circle upwards in a radius of 15 km. If the pilot has a mass of 90 kg and the aircraft is traveling at 686 m/s.

A) what will be the pilot's apparent weight (force normal)?



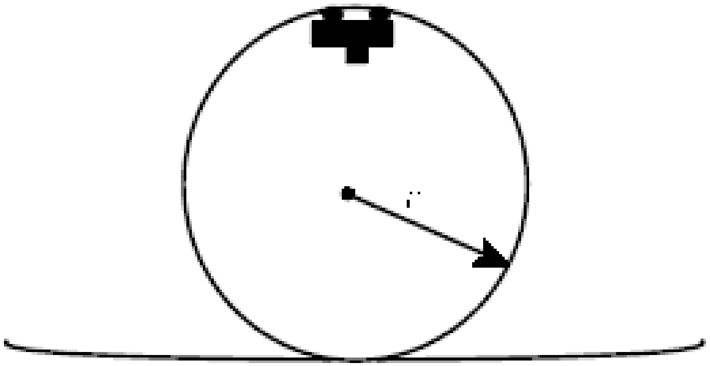
B) How many "g forces" does the pilot experience?

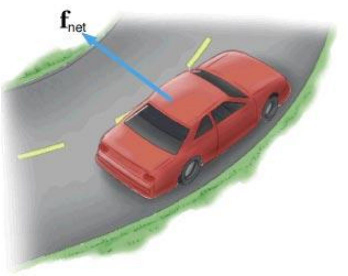
g force is how many times is the force of gravity that of the apparent force



Example 8

The diagram below represents the loop of a roller coaster. If the radius of the loop is 12.0 m, what is the minimum speed, at the top of the loop, required to prevent passengers from falling out? (public June 04)





Activity

Questions:

- Do #1-4 page 214
- Review Sheet
- Do Lab 2-1 pg 228