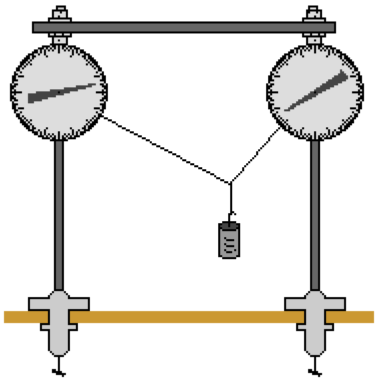
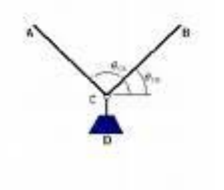
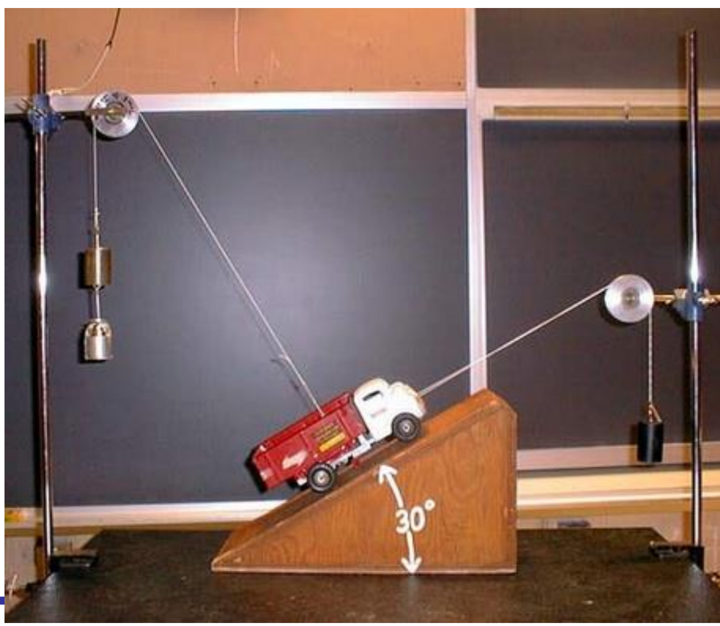
Physics 3204
Unit 1:
Section 4: Static Equilibrium

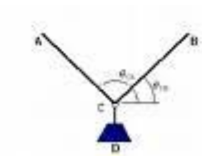




- **Section 4:**

Topic 1: Introduction to Static Equilibrium

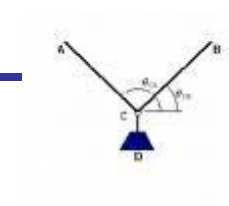




Keeping objects at rest is important in physics.

Statics refers to the study of keeping objects in a state of rest by applying forces on them

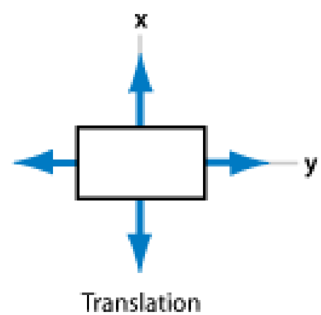
In order for an object to stay at rest, the net force on the object must be zero and the acceleration is zero. This means that all of the forces must balance each other out



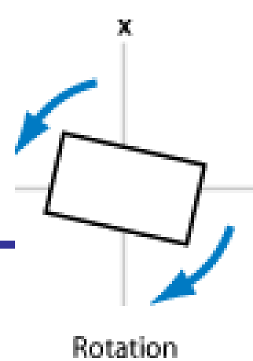
STATIC EQUILIBRIUM means stationary or at rest.

We will consider the following types of movement:

1) Translational Movement: uniform motion of a body in a straight line

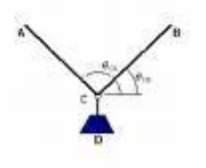


2) Rotational Movement: the action or process of rotating on or as if on an axis or center

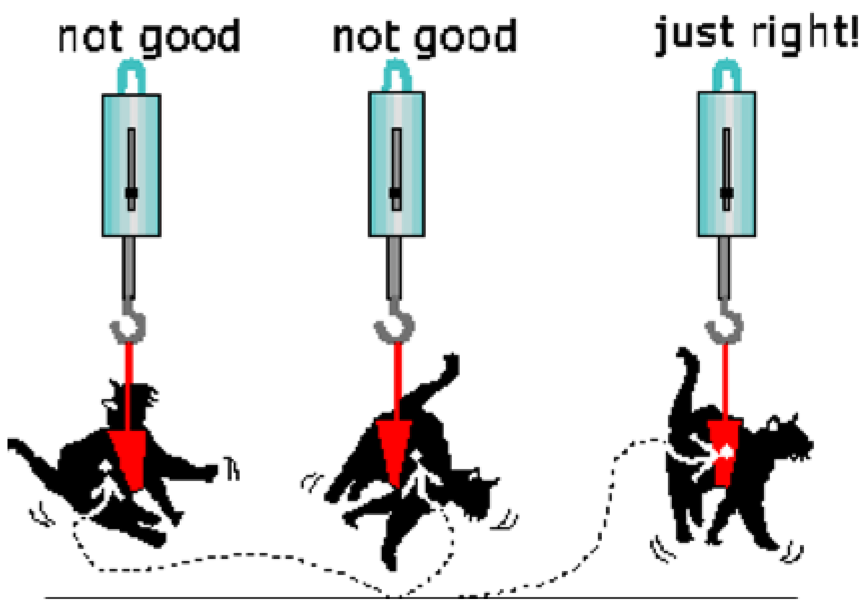


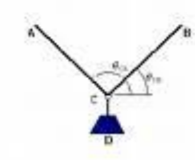
THE CENTRE OF MASS P233-234

- The study of static equilibrium is dependent on the centre of mass. We must consider forces on objects that not only cause them to "translate" but to "rotate" as well. To simplify these situations, we will assume that the mass of the object is located at its centre of mass. The centre of mass is also commonly referred to as the centre of gravity. Depending on the shape of the objects, the center of mass may be in the geometric centre or at some other location. See page 233 of your text for diagrams.



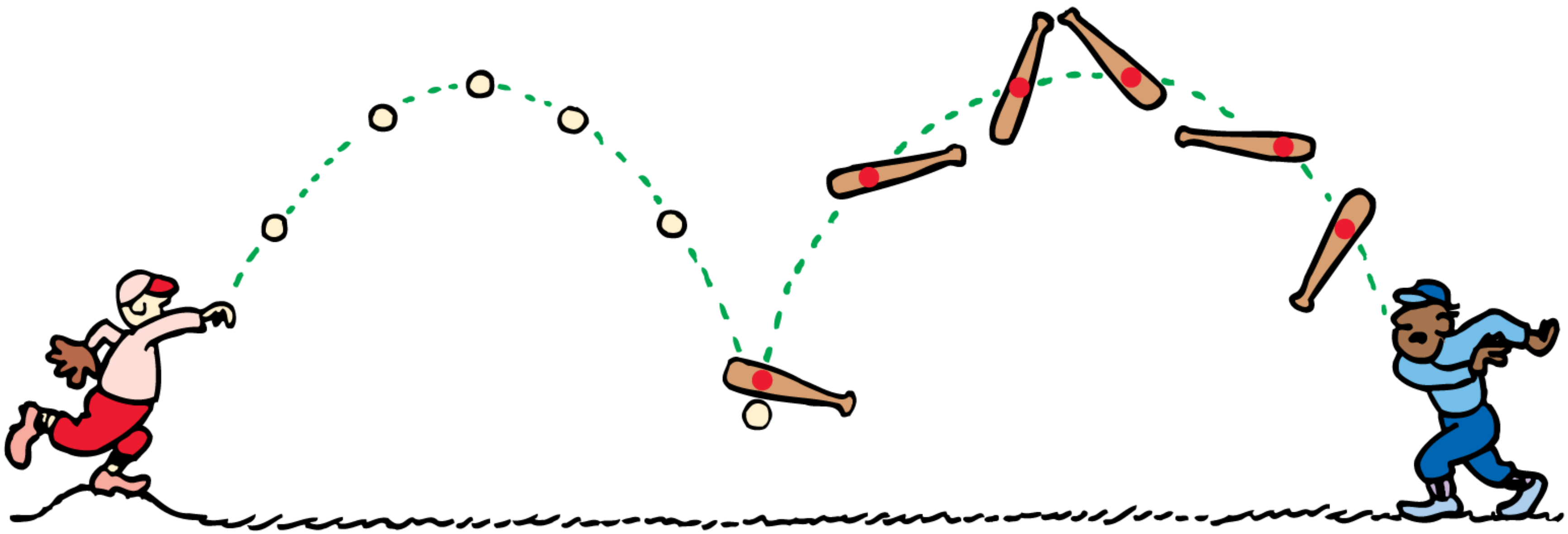
•Wherever the centre of mass is located, the force of gravity on each side of the centre of mass (gravity) is equal. We can determine the centre of mass of an object by hanging the object from three different points and following the procedure on page 233 and 234 of your text. Be sure to read this section.





11.3 Center of Mass

The centers of mass of the baseball and of the spinning baseball bat each follow parabolic paths.

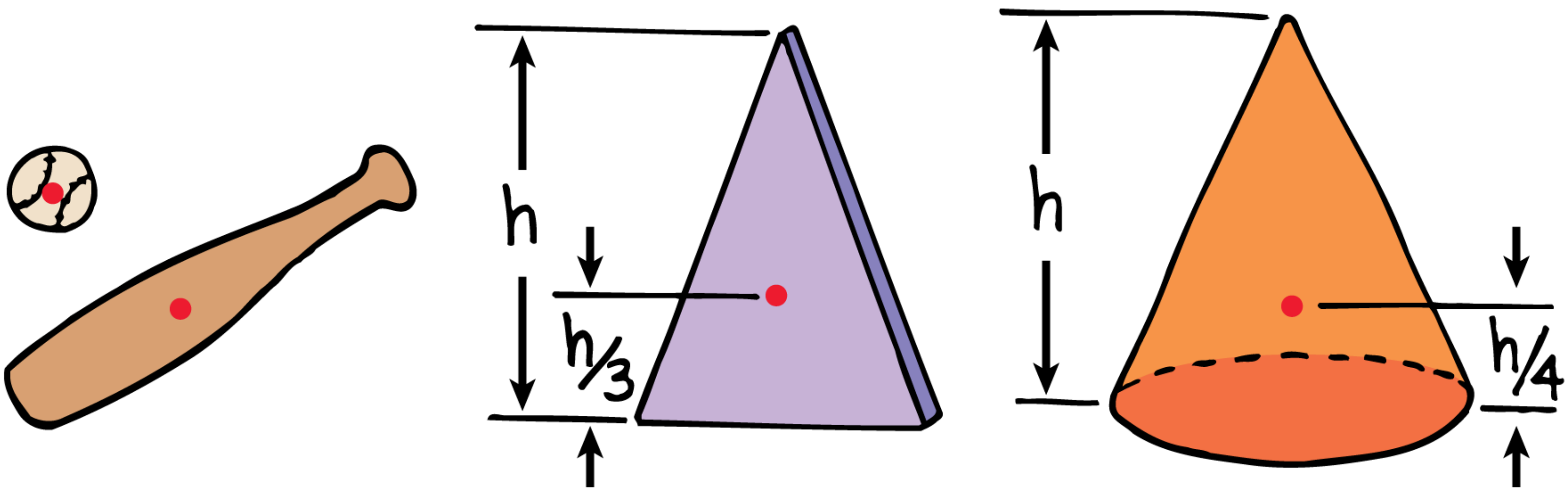


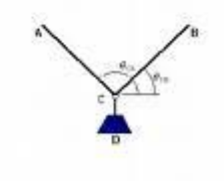
Location of the Center of Mass

For a symmetrical object, such as a baseball, the center of mass is at the geometric center of the object.

For an irregularly shaped object, such as a baseball bat, the center of mass is toward the heavier end.

The center of mass for each object is shown by the red dot.



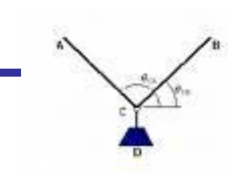


11.4 Center of Gravity

Center of mass is often called **center of gravity,** the average position of all the particles of *weight* that make up an object. For almost all objects on and near Earth, these terms are interchangeable.

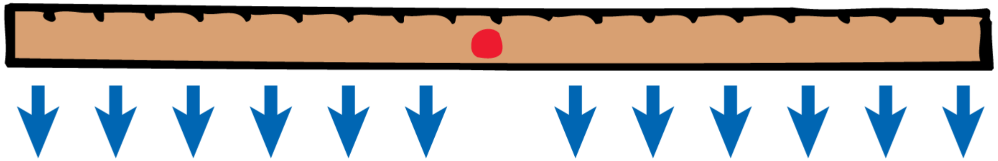
There can be a small difference between center of gravity and center of mass when an object is large enough for gravity to vary from one part to another.

The center of gravity of the Sears Tower in Chicago is about 1 mm below its center of mass because the lower stories are pulled a little more strongly by Earth's gravity than the upper stories.



11.4 Center of Gravity

The weight of the entire stick behaves as if it were concentrated at its center. The small vectors represent the force of gravity along the meter stick, which combine into a resultant force that acts at the CG.



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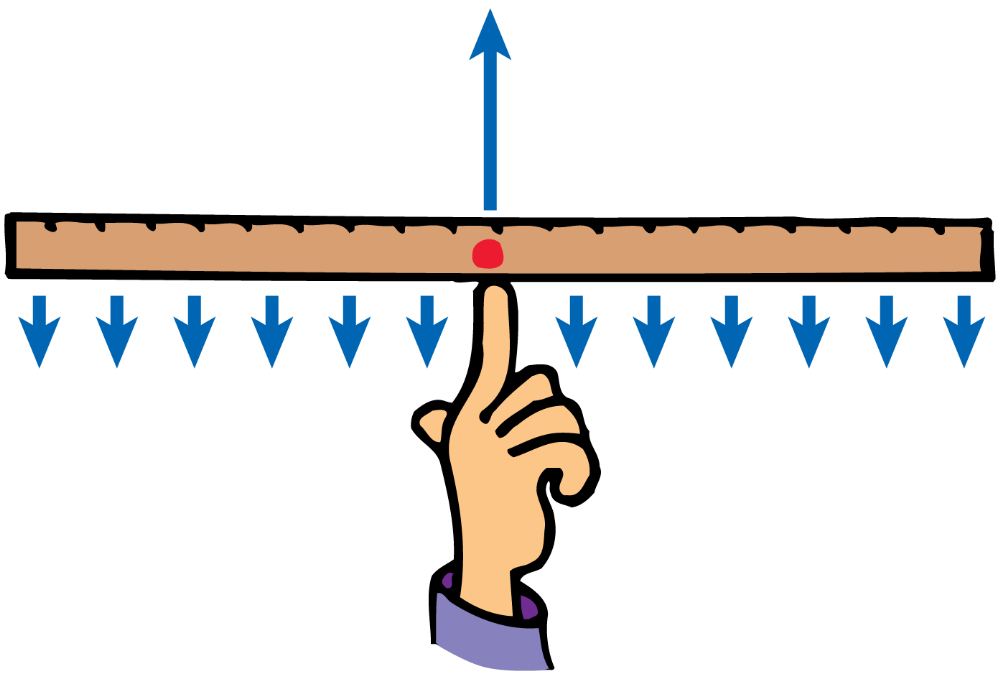


11.4 Center of Gravity

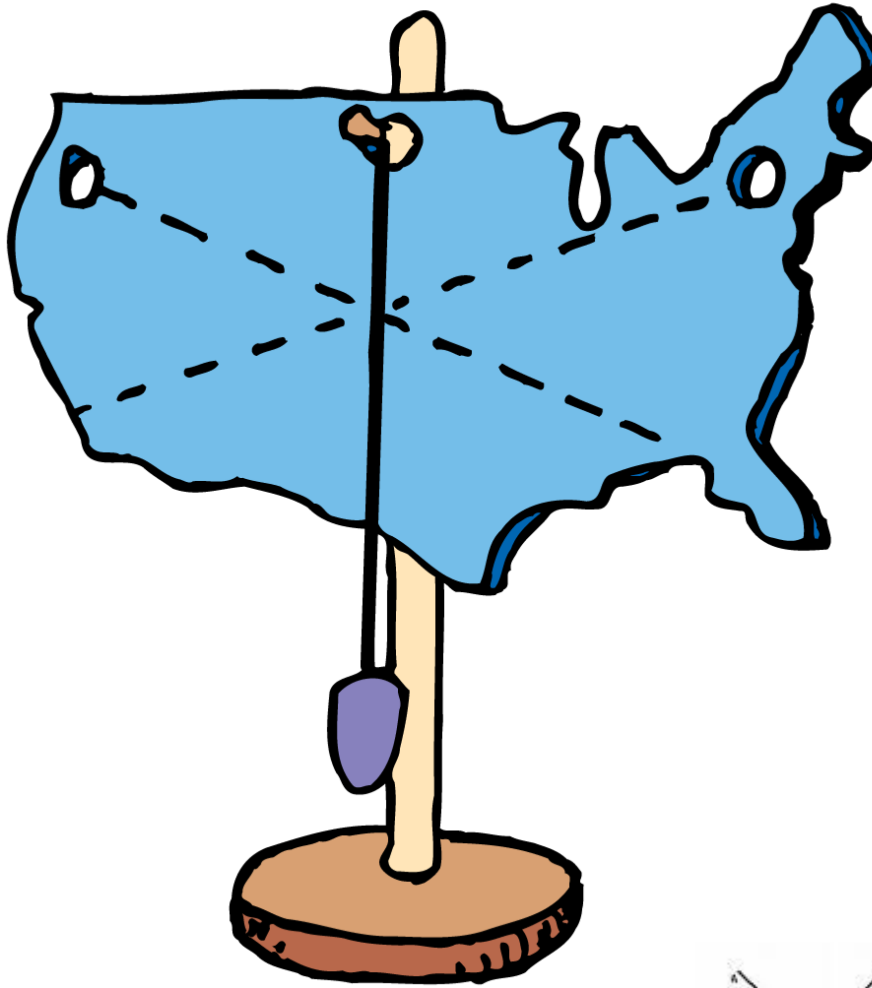
If you suspend any object at a single point, the CG of the object will hang directly below (or at) the point of suspension.

To locate an object's CG:

- Construct a vertical line beneath the point of suspension.
- The CG lies somewhere along that line.
- Suspend the object from some other point and construct a second vertical line.
- The CG is where the two lines intersect.

11.4 Center of Gravity

You can use a plumb bob to find the CG for an irregularly shaped object.



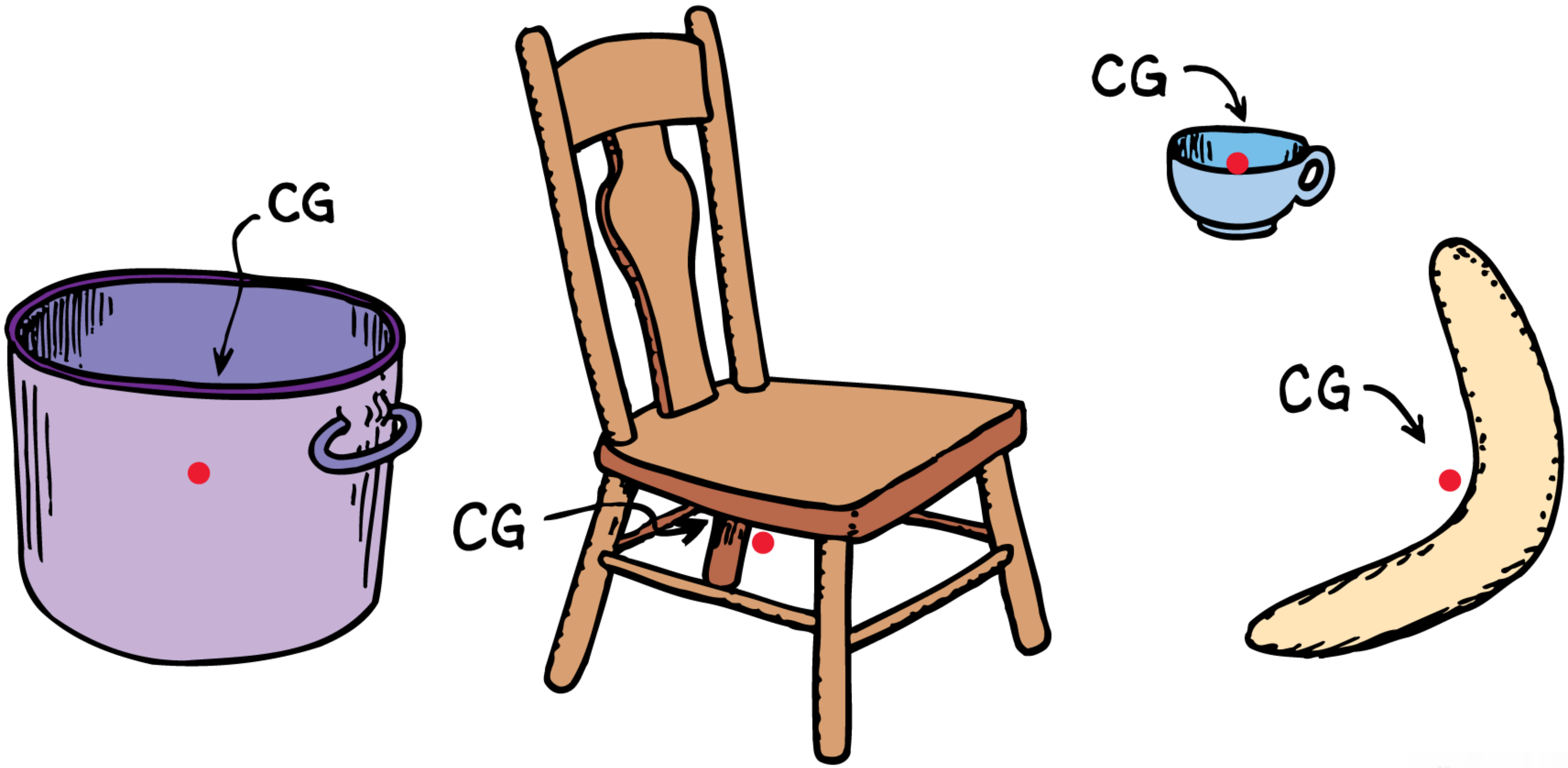
11.4 Center of Gravity

The CG of an object may be located where no actual material exists.

- The CG of a ring lies at the geometric center where no matter exists.
- The same holds true for a hollow sphere such as a basketball.

11.4 Center of Gravity

There is no material at the CG of these objects.



11.4 Center of Gravity

think!

Where is the CG of a donut?

11.4 Center of Gravity

think!

Where is the CG of a donut?

Answer:

In the center of the hole!

11.4 Center of Gravity

think!

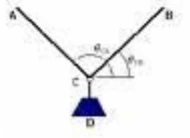
Can an object have more than one CG?

- **Section 4:**

Topic 2:

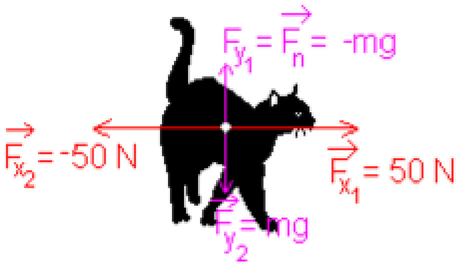
Translational

Equilibrium



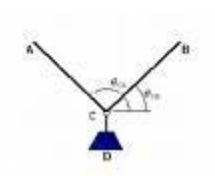
Example 1:

A cat attempt to escape by pushing with his hind legs. If the cat pushes with a force of 50 N to escape, What force must you apply so there will be no translational movement?



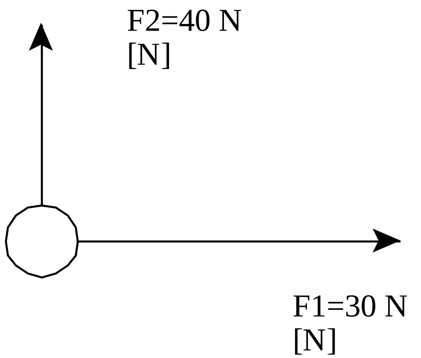
Read: (do not write)

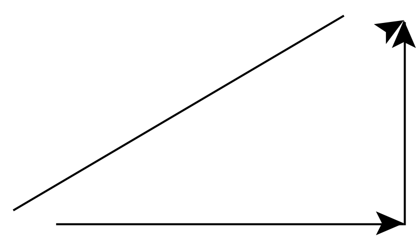
If the cat pushes straight forward, then to keep the cat still, you must pull backward with exactly the same force. When the cat is still because the two forces balance each other, we say that the cat is in static equilibrium and there will be no translational movement of the cat (no straight line movement). He will be at rest with no velocity and no acceleration because you are exerting a cancelling force force, the equilibrium force (Fe).



Example 2

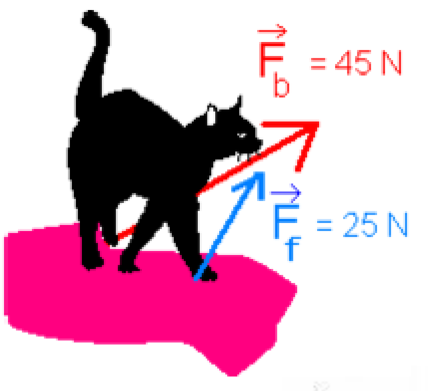
During a hockey game, three players simultaneous exert horizontal forces on the puck. While the forces are applied, the puck does not move (it is in equilibrium). Player one exerts a force of 30 N [E]. Player 2 exerts a force of 40 N [N]. What force must the third player be exerting to keep the puck motionless?

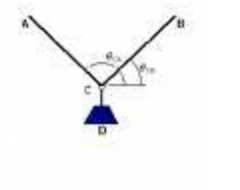




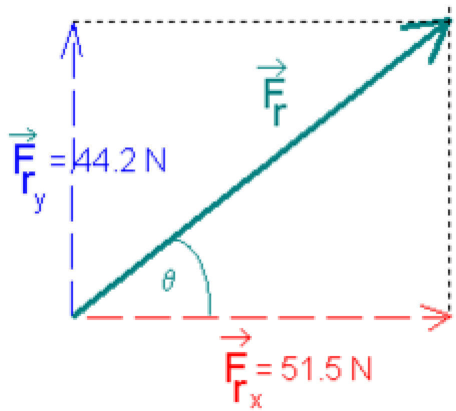
Example 3:

cat is pushing with its front legs and back legs. The front legs cause a forward force of F_f = 25 N [60º above the horizontal], and the back legs cause a forward force of F_b = 45 N [30º above the horizontal]. What must be the magnitude and direction of the equilibrium force (F_e) applied by you in order to maintain static equilibrium?





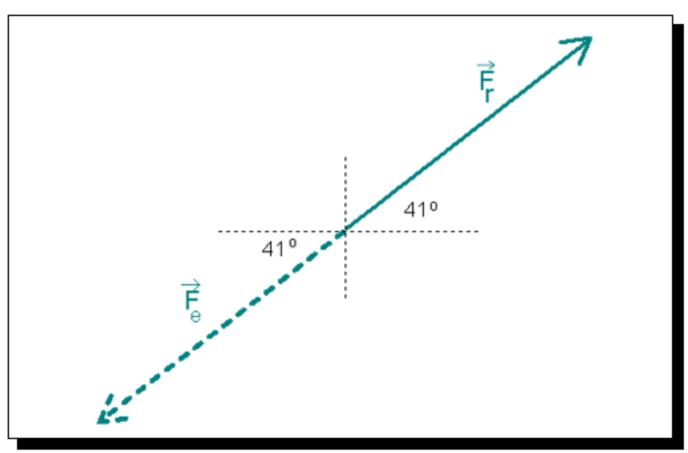
ΣF_x and ΣF_y are the x and y components of the resultant vector F_r.



$$\vec{F}_r = \sqrt{(\vec{F}_{r_x})^2 + (\vec{F}_{r_y})^2}$$
$$= \sqrt{(51.5)^2 + (44.2)^2}$$
$$= 67.9 N$$

$$\theta = \tan^{-1}\left(\frac{44.2}{51.5}\right)$$
$$\theta = \tan^{-1}(0.858)$$
$$\theta = 41^{o}$$

the resultant vector is 68 N [41º above the horizontal] or 68 N [41º north of east]. In order for you to cause static equilibrium of your cat, you must apply an equilibrant force exactly opposite to F_r. This will be Fe = 68 N [41_o south of west] as shown below

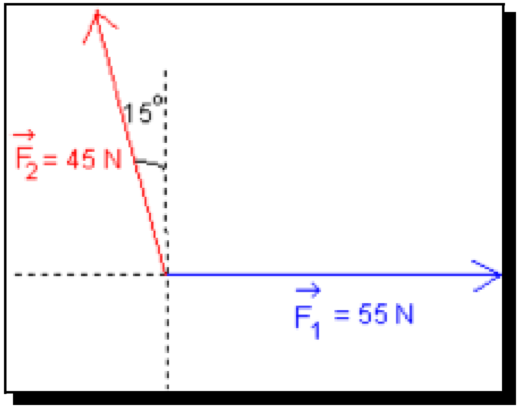


Example 4

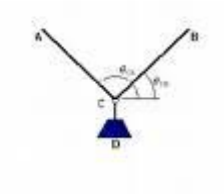
The following set of forces act on a common point:

F_1 = 55 N [due E]

F_2 = 45 N [N 15^o W]

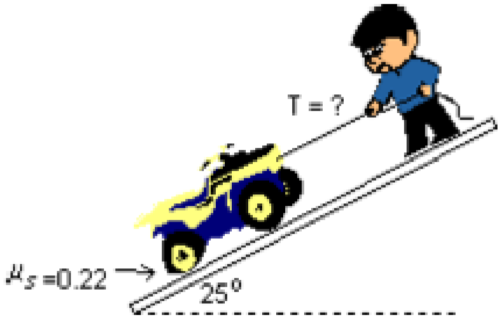


The direction of the resultant force (F_r) can be estimated to be "roughly in the northeasterly direction." Which means the equilibrant force (F_e) is in the opposite direction.



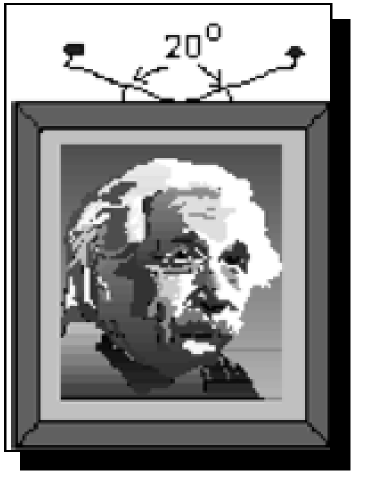
Example 5:

With the aid of friction caused by a flat tire Johnny just manages to hold the machine steady. The coefficient of static friction is 0.22 and the ATV has a mass of 250 kg. What is the tension in the rope?



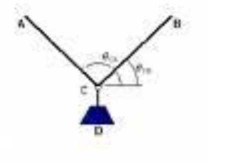
Example 6:

In the picture below the Center of Gravity of the piece of art is directly under the point where the supporting strings are attached. The hanging picture weighs 72 N and each of the supporting strings makes an angle of 20^o with the frame. What is the tension in each string?



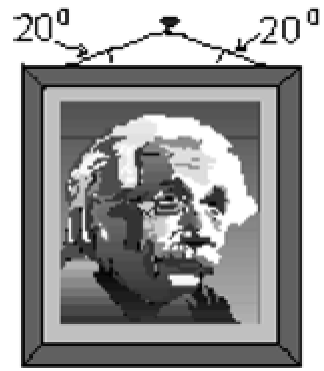
Note:

Because they make the same angle with the picture, the tensions (T_1 and T_2) in the strings equal each other.



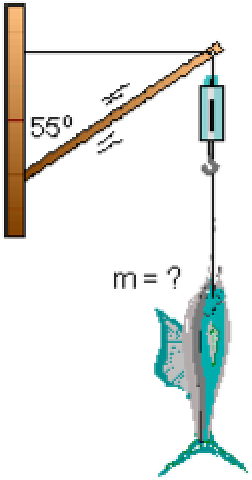
Example 7

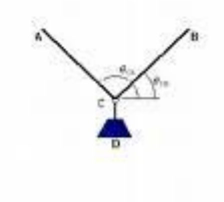
In the picture to the right the CG of the piece of art is directly under the nail that supports it. The hanging picture weighs 72 N and the supporting string forms an isosceles triangle with the frame as shown. What is the tension in the string?



Example 8:

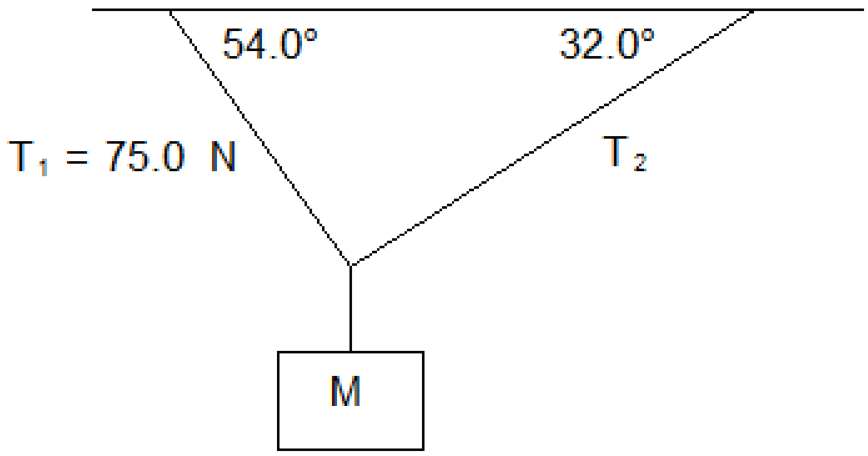
A boom set-up for weighing very large fish is shown in the picture. The boom can withstand a compression force of 3.0×10^3 N. What is the mass of the largest fish that can be weighed?





Example 9:

A sign of mass M hangs from two cables as shown below. Calculate the mass of the sign if it is in static equilibrium. **JUNE 2009**



Activity

Questions:

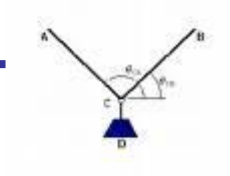
- PAGE 237 #1, #2
- PAGE 237, #3, #4, #5, AND #6
- PAGE 238--#6
- PAGE 267--#17 - #22
- PAGE. 268--#23 - #27

- **Section 4:**

Topic 3:

Rotational

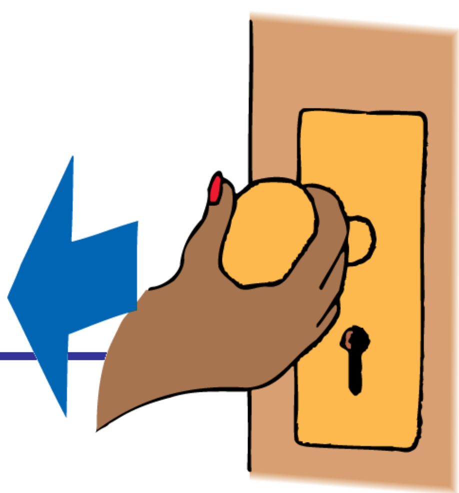
Equilibrium

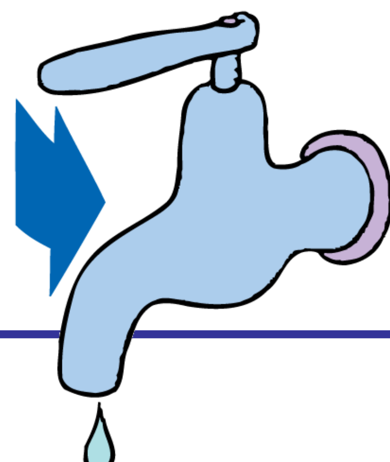


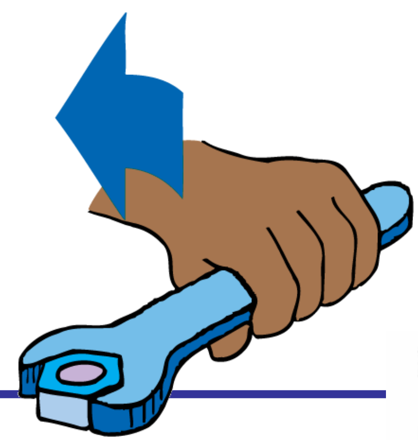
ROTATIONAL EQUILIBRIUM

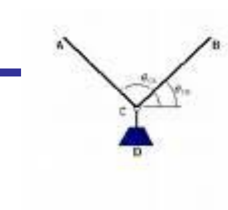
- So far we have seen what happens to an object when a force acts through the center of mass. However, a force may not necessarily act through the center of mass of an object.

If a force is applied to an object not through its center of mass, the mass has a tendency to rotate as opposed to translate. The rotational effect caused by a force is called torque · τ (Greek letter tau). Another name for torque is called the moment of force.



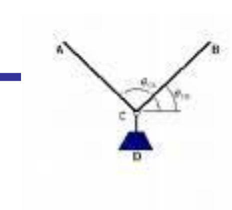






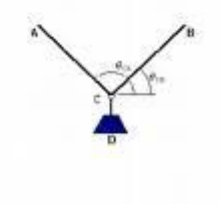
If the football is kicked in line with its center, it will move without rotating

A force must be applied to the edge of an object for it to spin.
If it is kicked above or below its center, it will rotate.



In order for an object to rotate, we must find the pivot point of the rotation. In addition, to calculate the torque on an object, we need the radius of the rotation and the perpendicular force that is applied to the object. In other words,

Torque can be calculated by finding the product of this perpendicular force and the radius.



Torque

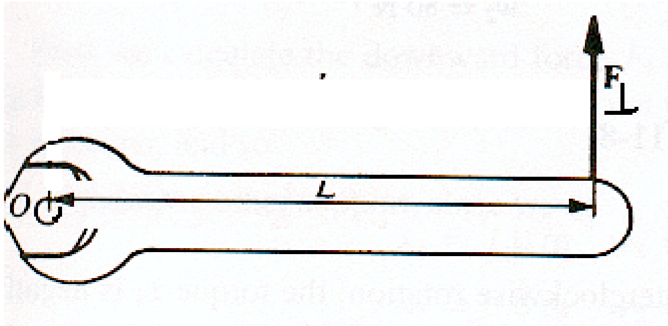
In order to calculate the torque on an object, you need the following:

1) **Find the pivot point of the rotation**

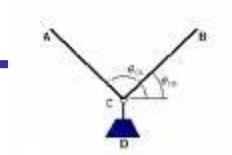
2) **The radius of the rotation**

3) **the perpendicular force that is applied to the object**

$$\vec{\tau} = r \cdot F_{perpendicular}$$



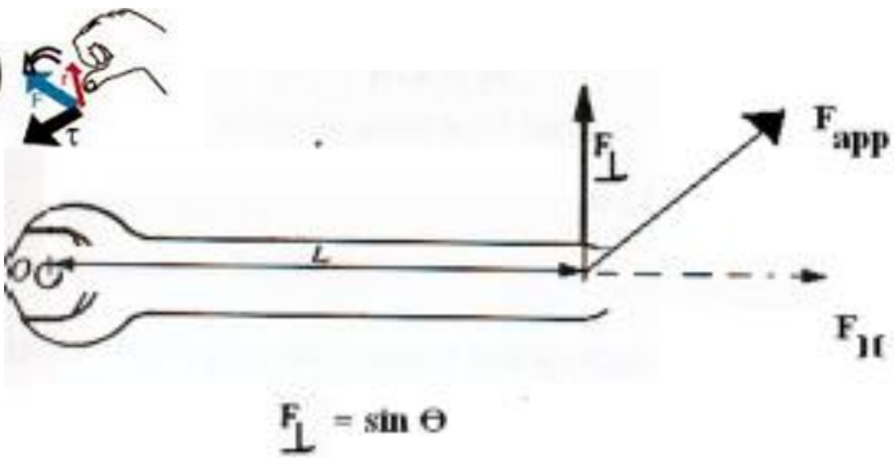
$F_{app=} = F_{\perp}$



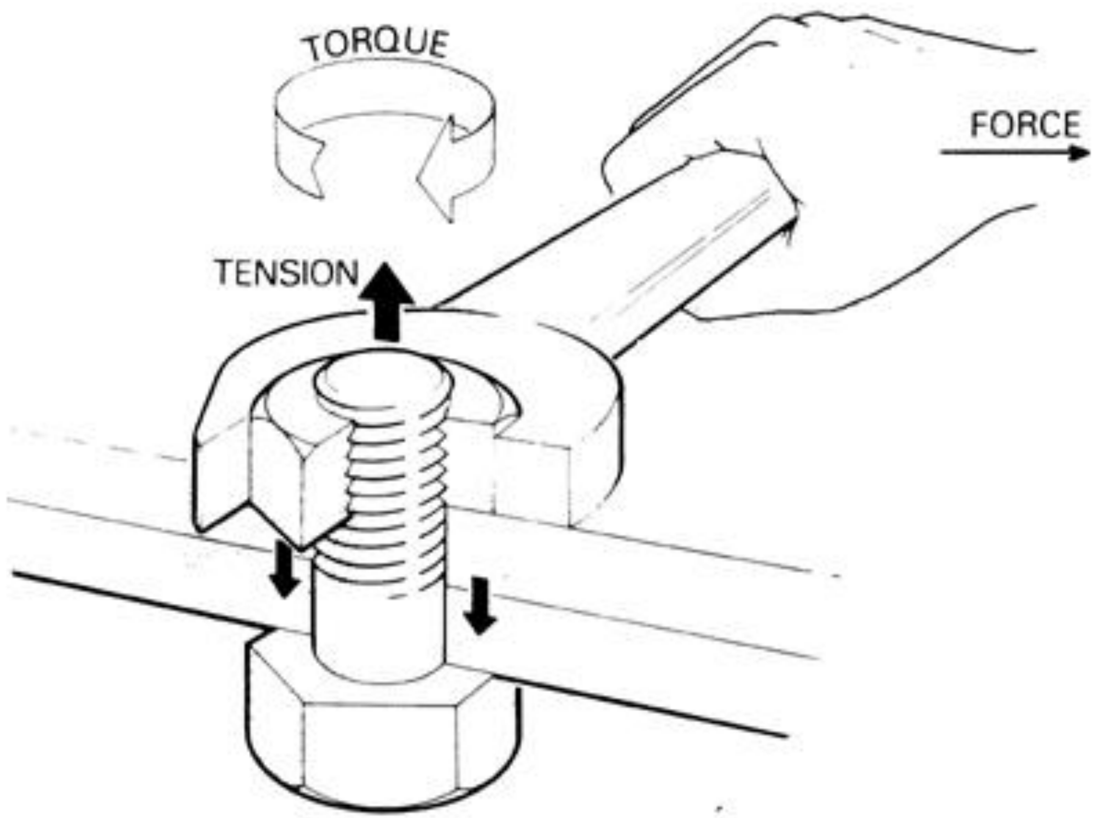
If a force causes an object to rotate and the force is not at a right angle to the radius, we must determine the perpendicular component of the force.

If the angle θ is the angle between the surface and the applied force, then the equation can be rewritten as

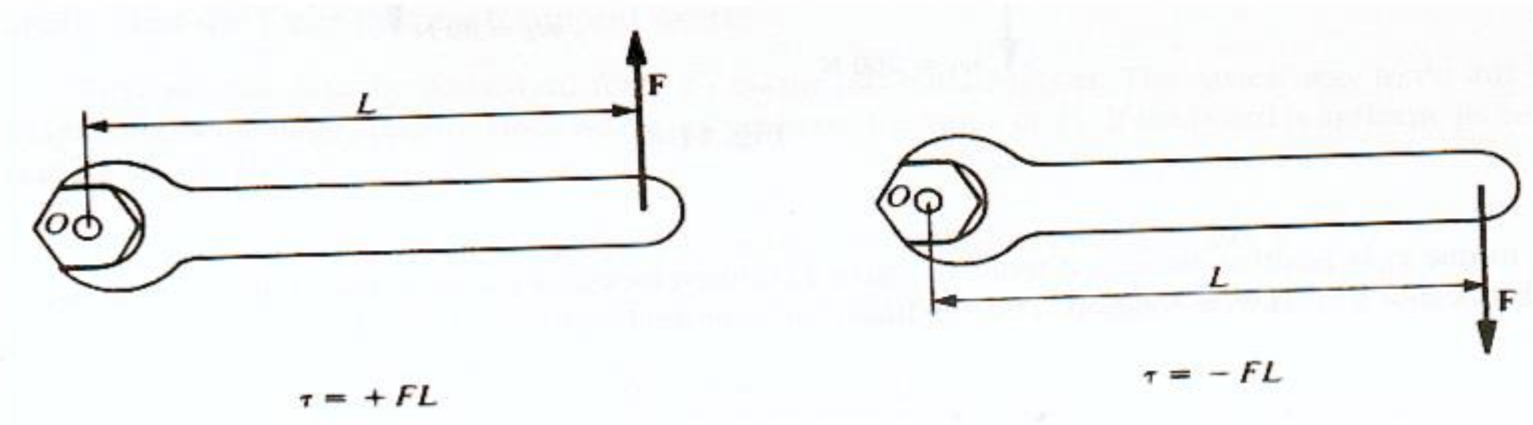
$$\text{Torque} = \tau = r\,F\sin\theta$$



HOW WOULD YOU INCREASE TORQUE ?



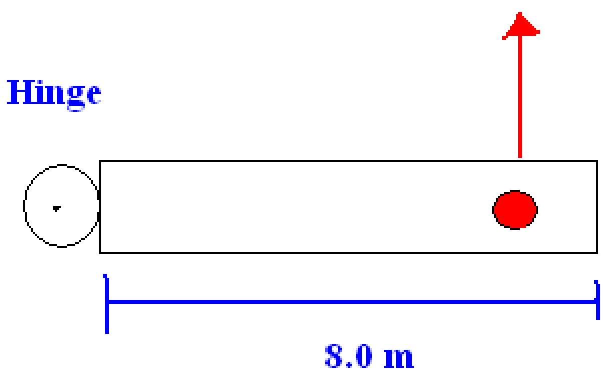
One suggestion that you may consider is that since torque is a vector, we should attach a direction system to the situation. Make your rotations consistent, in other words, you could make all clockwise rotations one sign (+ or -) and all counterclockwise rotations the opposite



Example 1:

A door knob is located 1.0 m from the hinge (Pivot point or the axis of rotation) and a person closes the door by applying a 30 N of force.

A) Find the τ if the F is perpendicular

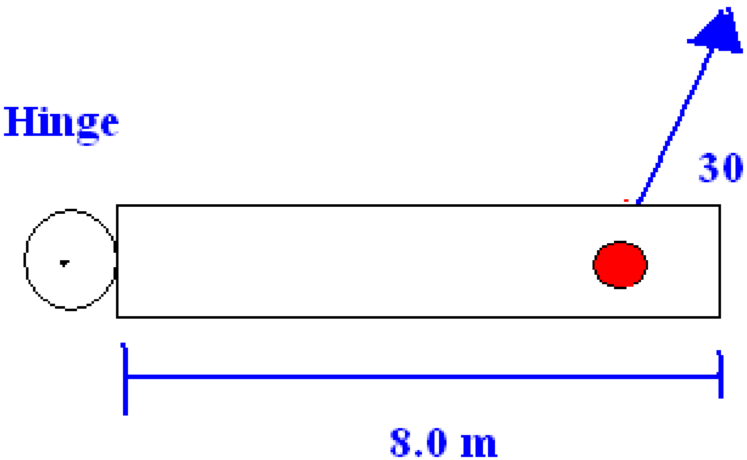


$$\tau = F_{app} Sin\theta \cdot r$$

$$= (30N)(\sin 30)(1.0m)[$$

$$= 15N \cdot m[ccw]$$

B) Find the τ if the F is at a 30 degree angle



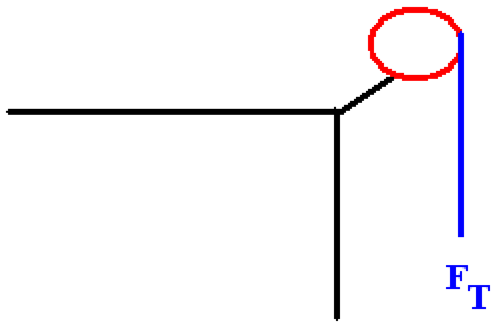
$$\tau = F_{appr} \cdot r$$

$$= (30N)(1.0)[$$

$$= 30N \cdot m[ccw]$$

Example 2:

A student pulls down with a force of 40 N on a rope that winds around a pulley of radius 5.0 cm. What is the force of this force?



$$\tau = F_{app} r$$

$$= (40N)(0.050m)[$$

$$= 2.0 N \cdot m[cw]$$

Force is perpendicular to the pulley

For any object to be in static equilibrium, the net clockwise torque about any point must equal the net counterclockwise torque about that point.

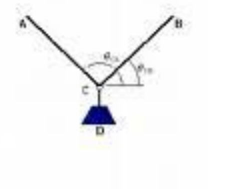
$$\tau_{cw} = \tau_{ccw}$$

Therefore, the three condition for static equilibrium:

$\Sigma f_x = 0$ (Translational movement)

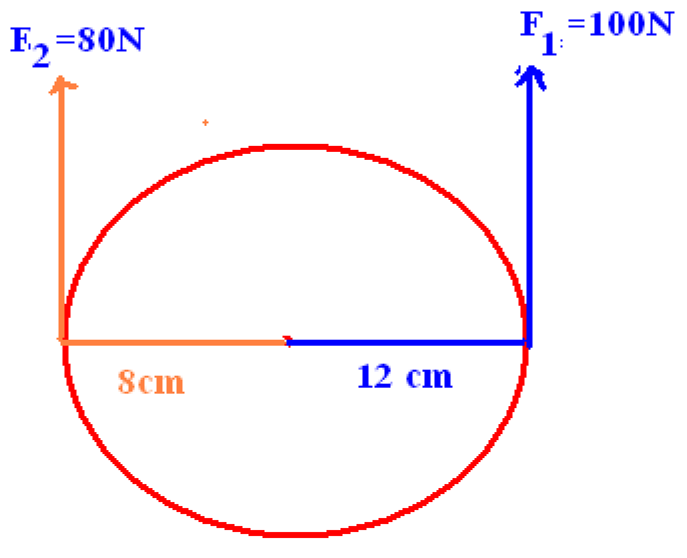
$\Sigma f_y = 0$ (Translational movement)

$\tau_{net} = 0$ (Rotational movement)



Example 3:

What is the net torque on the cylinder shown?

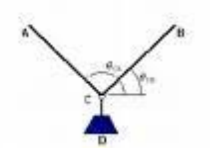


$$\tau_{net} = \tau_{cw} + \tau_{ccw}$$

=(-80N)(0.080m)+ (100N)(0.12m)

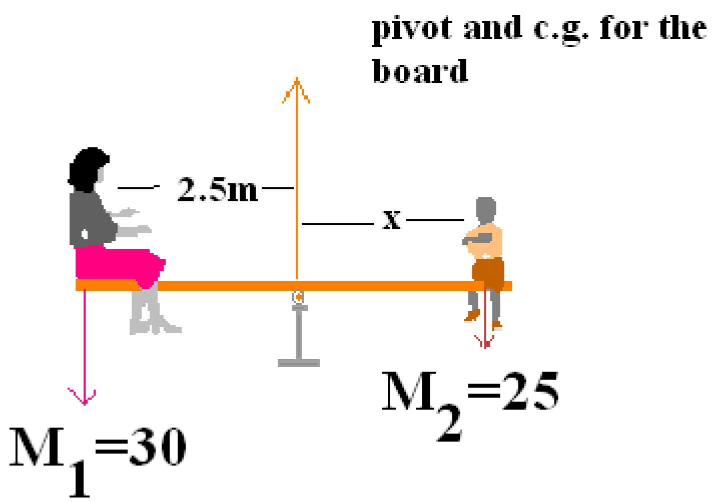
=-6.4 N m [cw]+12 N m [ccw]

=5.6N .m [ccw]



Example 4:

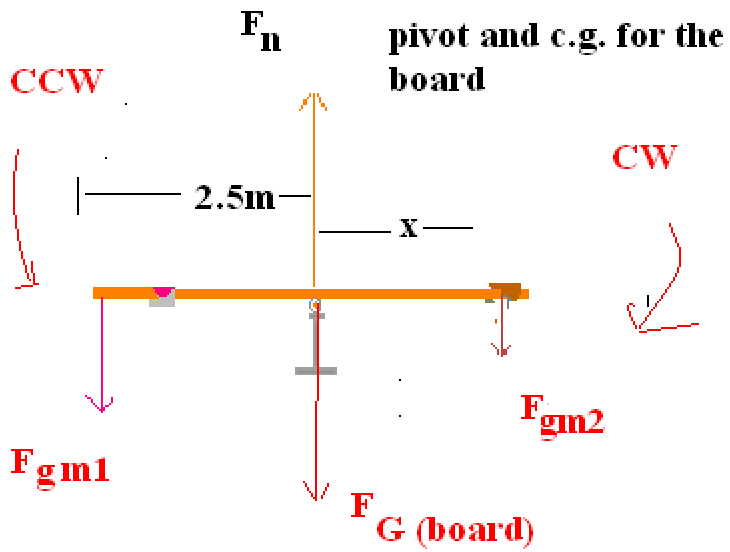
A 2.0 kg board serves as a seesaw for two children. One child has a mass of 30 kg and sits 2.5 m from the pivot point (Fulcrum)(Note: his c.g is 2.5 m from the pivot point). At what distance x) from the pivot must a 25 kg child place herself to balance the seesaw? Assume the board is uniform and centered over the pivot point

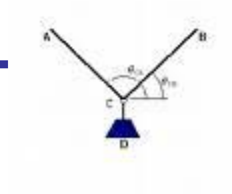


NOTE:

- For a uniform board the c.g. is in the centre
- only use force acting on the rigid bodies
- Pivot point not included in the calculations sine r=0m, thus $\tau = 0$

FBD





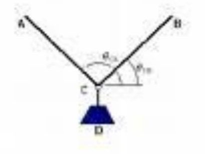
$$\tau_{cw} = \tau_{ccw}$$

$$Fg_{m2} \bullet x = Fg_{m1}(2.5)$$

$$x = \frac{(Fg_{m1)} \bullet (2.5)}{Fg_{m2}}$$

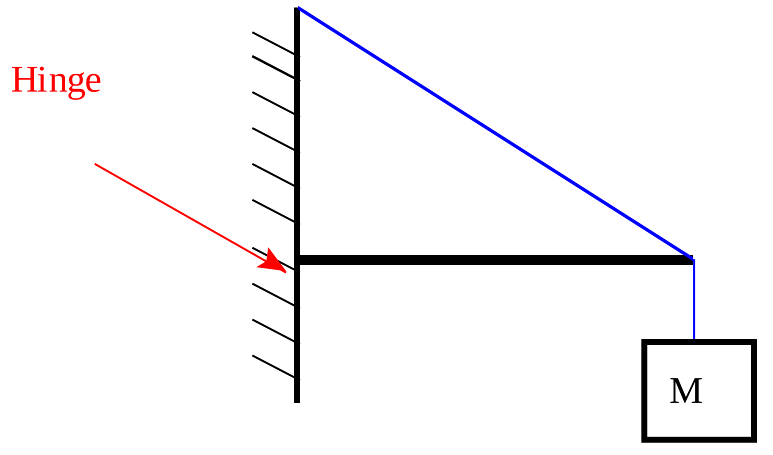
$$\frac{(30)(9{,}80)(2.5)}{(25)(9.80)}$$

$$3.0m$$



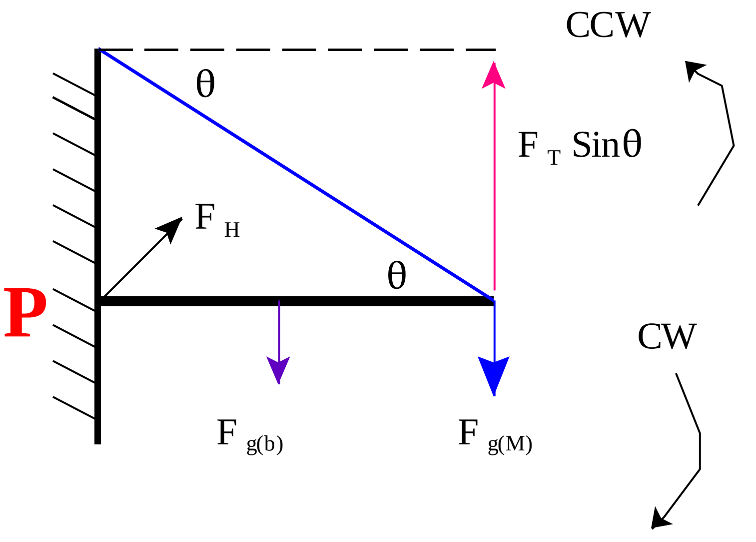
Example 5:

A uniform bean (C.M is in the center) 2.20m long with m=25.0 kg, is mounted by a hinge on a wall as shown. The beam is held in a horizontal position by a wire that makes an angle $\theta = 30.0^o$. The beam supports a mass, M_1 of 280 kg suspended from its end.



A) Find the force of the tension (FT) in the supporting wire

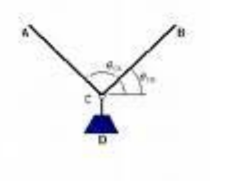
Sketch FBD



F_H = force of the hinge on the beam

P= Pivot Point

$F_{g(b)}$ = force of gravity on the beam



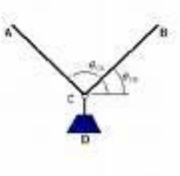
$$\tau_{cw} = \tau_{ccw}$$

$$\left(F_{gb}\right)\left(\frac{1}{2}r\right) + \left(F_{gM}\right)(r) = F_T(\sin 30)(2.20m)$$

$$(25)9.80)(1.10) + (280(9.80)2/20) = F_T(\sin 30)(2.20m$$

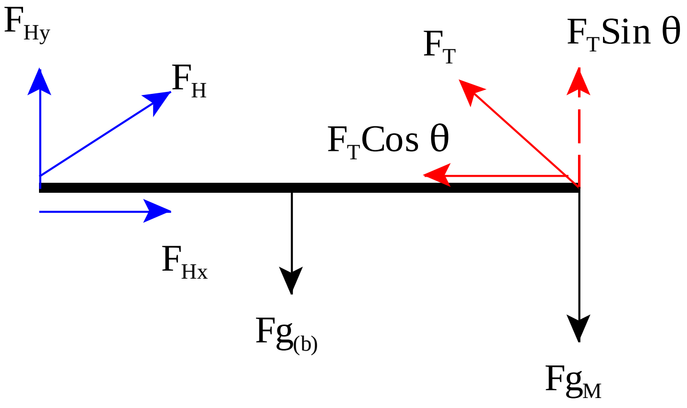
$$= 5733N$$

$$= 5.73 \times 10^3 N$$



B) Find the (x) and (y component of the force of the hinge and the beam.

FBD:



At Rest : $\Sigma F_x = 0$ and $\Sigma F_y = 0$

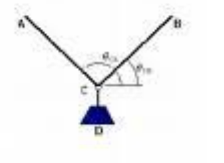
X- direction

$\Sigma F_x = 0$

$F_{Hx} + F_T \cos\theta = 0$

$F_{Hx} - F_T \cos\theta = 0$ (direction system

$F_{Hx} = F_T \cos\theta$
$= (5.73 \times 10^3\ N)\cos 30$
$= 4.96\ \times 10^3\ N$ [right]



Y - direction

$\Sigma F_y = 0$

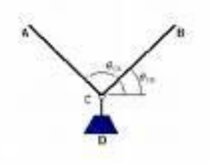
$F_{Hy} + F_T \sin\theta + F_{gb} + F_{gM} = 0$

$F_{Hy} + F_T \sin\theta - F_{gb} - F_{gM} = 0$

$F_{Hy} + F_T \sin\theta = F_{gb} + F_{gM}$

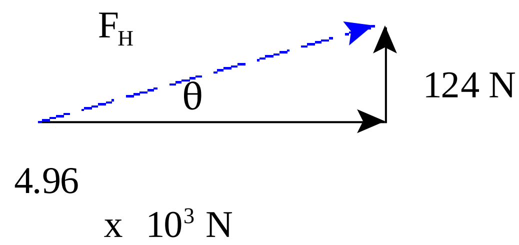
$F_{Hy} += F_{gb} + F_{gM} - F_T \sin\theta$

$= 124$ N [up]



C) Find the magnitude and direction of the force exerted by the pivot on the bean.

Sketch



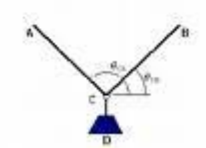
$F_H{}^2 = (4.96\ \text{x}\ 103\ \text{N})^2 + (124)^2$

$= 4.96\ \text{X}\ 10^3\text{N}$

$\tan\theta = \frac{0pp}{adj}$

$= \frac{124}{4.96x10^3}$

$\theta = 1.4$

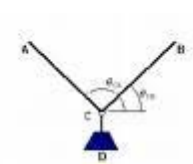


F_H = 4.96×10^3 N [right 1.4 up] not correct

= 4.96×10^3 N [1.4 above the (+) x axis]

Or

4.96×10^3 N [1.4 above the horizontal]



Example 6

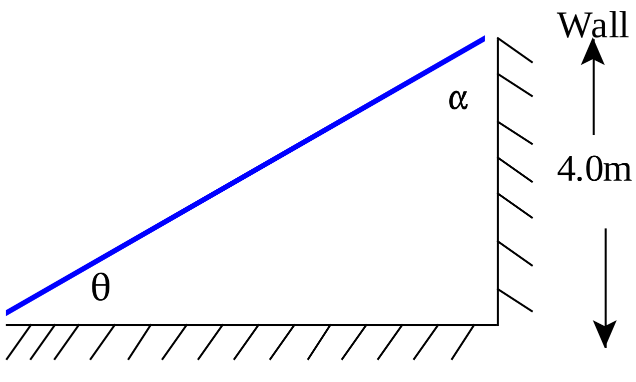
A 5.0 ladder leans against a wall at a point 4.0 above the ground as shown. The ladder is uniform and has a mass of 12 kg. Assuming the wall is frictionless (but not the ground) determine:

A) the force of the wall on the ladder

B) The force of the friction keeping the ladder from slipping

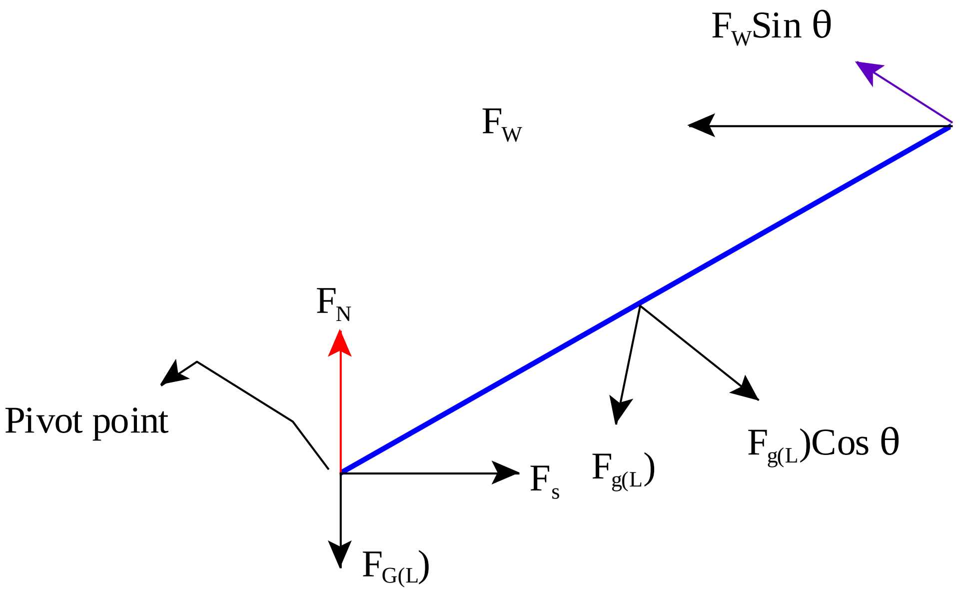
C) the coefficient of static Friction

Sketch



α= alpha

FBD



Step 1: Calculate the angle the ladder makes with the horizontal (floor)

A)

$\sin\theta = 4/5$

$\theta = \sin^{-1}(4/5)$

$\theta = 53^{o}$

Step 2: Find $\Sigma\tau_{net}=0$

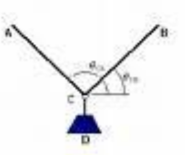
$$\tau_{cw} = \tau_{ccw}$$

$$\left(F_{g(L)}Cos\theta\right)\left(\frac{1}{2}r\right) = F_w(\sin\theta)(r)$$

$$(12)(9.80)(\cos 53)(2.5)) = F_w(\sin 53)(5)$$

$$177\,Nm = (Fw)3.99$$

$$Fw = 44N$$



b) if the ladder is not slipping then $F_S = F_w$ or

$\Sigma F_x = 0$

$F_s - F_w = 0$

$F_s = F_w = 44N$

C)

We know that $F_s = \mu F_n$

$\mu = F_s/F_n$ (hence, we must find Fn)

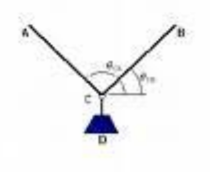
$\Sigma F_y = 0$

$F_n - F_{G(L)} = 0$

$F_n = F_{G(L)}$

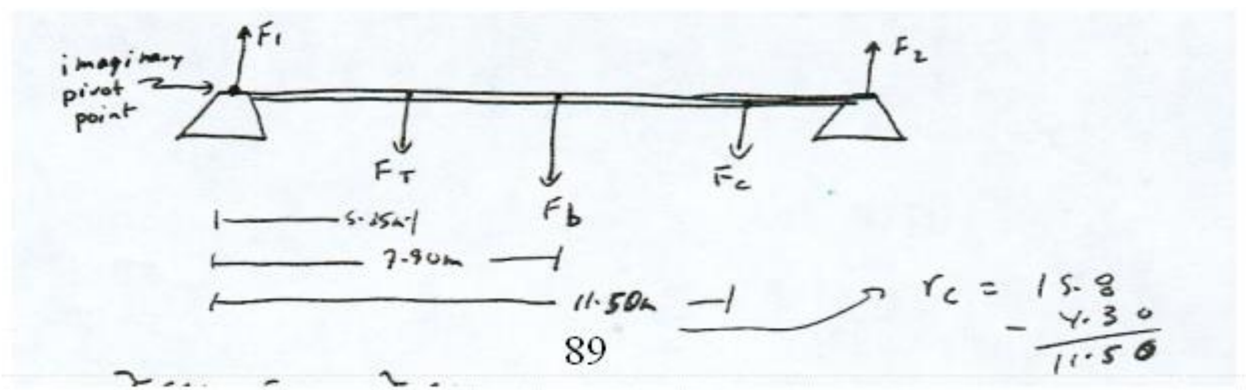
$F_n = (9.80)(12)$
$= 118$ N or 1.2×10^2 N

$\mu = 44\text{N}/1.2 \times 10^2\text{N} = 0.37$



Example 7

A uniform wooden bridge is 15.8 m long and weighs 1.24 x 10^5N. It is supported by a pillar at each end. How is the weight shared by the pillars when there is 2.88 x10^4 N truck 5.25 m from one end and a 1.50 x 10^4 car 4.30 from the other end?



Summary of Equilibrium

An object is in “Equilibrium” when:

1. **There is no net force acting on the object**
2. There is no net Torque *(we’ll get to this later)*

In other words, the object is **NOT experiencing linear acceleration** or rotational acceleration.

$$a = \frac{\Delta v}{\Delta t} = 0$$

$$\alpha = \frac{\Delta \omega}{\Delta t} = 0$$

We’ll get to this later