


## KINEMATICS FORMULAE:

|  | Equation | Variables found in equation | Variables not in equation |
| :---: | :---: | :---: | :---: |
| Equation 1 | $\dot{v}_{1}=\dot{v}_{1}+\dot{a}_{w v} \Delta t$ | $a_{e w}, \Delta t, v_{6}, v_{1}$ | $\Delta d$ |
| Equation 2 | $v_{1}^{2}=v_{1}^{2}+2 a_{w} \Delta d$ | $\Delta d, \mathrm{a}_{n \mathrm{~m}} \mathrm{v}_{1}, V_{1}$ | $\Delta t$ |
| Equation 3 | $\Delta \vec{d}=\overrightarrow{v_{1}} \Delta t+{ }_{2}^{1} \vec{a}_{x i} \Delta t^{2}$ | $\Delta d, \dot{a}_{\mathrm{am}} \Delta t, \dot{v}_{1}$ | $\dot{v}_{1}$ |
| Equation 4 | $\Delta \vec{d}=\vec{v}_{1} \Delta t-1_{2}^{1} \vec{a}_{n i} \Delta t^{2}$ | $\Delta \vec{d}, \vec{a}_{\text {ev }} \Delta t, \vec{v}_{t}$ | $\vec{v}$ |
| Equation 5 | $\Delta \dot{d}=\left(\frac{\dot{v}+\dot{v}}{2}\right) \Delta t$ | $\Delta \vec{d}, \Delta t, \vec{v}_{3}, \vec{v}_{1}$ | $\overrightarrow{a d o}^{\text {a }}$ |

REMEMBER: ----To change from km/h to m/s, you must DIVIDE BY 3.6
----To change from m/s to km/h you must MULTIPLY BY 3.6

NOTE: The sample problems in this lesson involve objects moving linearly in the VERTICAL plane or in the $Y$ direction. In other words, the objects are rising or falling in the Earth's Gravitational Field.


$$
\mathrm{g}=-9.8 \mathrm{~m} / \mathrm{s}^{2}
$$

## Sample Problem:

A baseball is "popped" straight upward, leaving a bat at $35 \mathrm{~m} / \mathrm{s}$
(a) How high does the ball rise in the air?
(b) How much time will the ball be in the air?
(c) What is the ball's position 5 seconds after it leaves the bat?
(d) How fast will the ball be moving and in what direction at the 5 second time interval?

## Activity



- Extra Read Text P.62-63
- Handout on "acceleration due to gravity
- Text Practice Problems: Pages 7374


Unit 1

INTRODUCTION TO PROJECTILE MOTION

Text Reference 3.2


# Fownot B I L L A M E N D 

Name: Peter Forx


Date: Not aa often as Sd dike to, sudly.


## EXAMPLES OF PROJECTILE MOTION



PROJECTILE MOTION: is the motion of an object fired or thrown at an angle to the horizontal whereby the only force acting on the object is GRAVITY. Also known as Parabolic Trajectory (Curved motion).

PROJECTILE:

TRAGETORY

A projectile is an object upon which the only force acting is gravity.
is the path a moving object follows through space as a function of time.

Thus, an object undergoing projectile motion, is moving forward (in the $x$ direction) as it would in outer space and also falling (in the y direction) at the same time due to the Earth's Gravitational Field.


## LET'S ANALYZE THE JUMP



## Projectile Motion OR 2D Kinematics Calculations:

The most common misconception in projectile motion problems is students forget that they are working with VECTORS

## Projectile motion is TWO DIMENSIONAL ( x and y motion).

The key to solving these problems is to break these two dimensional problems into two separate one dimensional parts and then recombine them to produce a final answer.

The problem set-up will have a set of givens in the X-direction and another set in the Y-direction.


The Equations used in solving projectile motion problems are the same as the equations you worked with in Physics 2204 when you studied Kinematics.

$$
\begin{array}{ll}
d=\underline{v_{1}}+\frac{v_{2}}{2} \times t & \\
V_{2=} V_{1}+a x t & V_{2 x}=V_{1 x}+a_{x} x t \\
d=V_{1} t+1 / 2 a t^{2} & d_{y}=V_{1 y}+1 / 2 a_{y} t^{2} \\
2 a d=V_{2}^{2}-V_{1}^{2} & \text { And so on...... }
\end{array}
$$

But now we will place $x$ and $y$ notation by the variables according to the dimension we are trying to find information on...either X or Y

Apply the quadratic formula to solve for " $\Delta \mathrm{t}$ ":
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
Calculations involving projectile motion require some elementary trigonometry:

$$
\begin{aligned}
& \text { sint } \theta=\frac{\text { opposite side }}{\text { hypotenuse }} \\
& \cos \theta=\frac{\text { adjacent side }}{\text { hypotenuse }} \\
& \text { tan } \theta=\frac{\text { opposite side }}{\text { adjacent side }}
\end{aligned}
$$

Projectiles follow a PARABOLIC path and travel in an ARC. For the purposes of your studying of projectile motion, the following will be considered:
.....air friction will be negligible
...in the X direction there is UNIFORM MOTION ( $\mathrm{a}=0 \mathrm{~m} / \mathrm{s}^{\mathbf{2}}$ )
... in the Y direction there is UNIFORM ACCELERATION
( $\mathrm{a}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$ )



Notice the ball lands back in the truck...
only if the truck moves with constant velocity
The ball lands in the truck because they both have the same Vx

## Timing a Parabolic Trajectory:


$A$ and $B$ have the same mass and are at the same height At the same time, $A$ is dropped while $B$ is thrown horizontally

Which ball will hit the ground first, $A$ or $B$ ?

## Watch the following Video

## THE ANSWER IS......



No matter how hard you throw B, BOTH WILL STAY IN THE AIR FOR THE SAME AMOUNT OF TIME and thus hit the ground at the same time!!!!!!!

TIME is the only things connecting the $X$ and $Y$ components for a projectile

$A$ and $B$ are falling at the same rate ( $9.8 \mathrm{~m} / \mathrm{s}$ every second)
The initial push only caused $B$ to travel further in the $x$ direction. B still fell the same vertical distance ( $d y$ ) at the same rate ( $a y=9.8 \mathrm{~m} / \mathrm{s}^{2}$ )

The VERTICAL and HORIZONTAL components are INDEPENDENT of each other.

Thus $A$ and $B$ have the same: vertical distance ( dy ) rate of acceleration ( $\mathrm{ay}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ) time to fall (t)

## RANGE

## RANGE refers to the horizontal displacement

$$
D_{x}=V_{x} t
$$



- Complimentary launch angles will travel the same horizontal distance (Range)!!!



## Maximum Range

- Imagine a cannonball launched from a cannon at three different launch angles - 30-degrees, 45-degrees, and 60degrees. The launch speed is held constant; only the angle is changed. Which cannon ball has the maximum range?
-The "hang time" of a projectile is determined by the initial value of the vertical velocity component. The smaller the initial value of Vy, the shorter the hang time.
-The range of a projectile is determined by two parameters - the initial value of the horizontal velocity component and the hang time of the projectile.

-The projectile launched at 45-degree has the ability to achieve the greatest range because it has both a good hang time and $V_{x}$.


Maximum range is at 45 degrees

Maximum flight time is 90 degrees

## Solving Problems Involving Projectile Motion

1. Read the problem carefully, and choose the object(s) you are going to analyze.
2. Draw a diagram.
3. Choose an origin and a coordinate system.
4. Decide on the time interval; this is the same in both directions, and includes only the time the object is moving with constant acceleration $g$.
5. Examine the $x$ and $y$ motions separately.

# 3-8 Solving Problems Involving Projectile Motion 

6. List known and unknown quantities. Remember that $v_{x}$ never changes, and that $v_{y}=0$ at the highest point.
7. Plan how you will proceed. Use the appropriate equations; you may have to combine some of them.

## SUMMARY

 called its trajectory.The trajectory of the ball shows the two components of the projectile motion namely $\mathrm{V} x$ and $\mathrm{V} y$.
$\mathrm{V} y$ ( y velocity vector ) gets progressively longer.
$\mathrm{V} x$ ( x velocity vector) is the same length throughout the motion
....distance in the $X$ direction is defined as the RANGE

$$
D_{x}=V_{x} t
$$

....TIME is constant in both the X and Y direction

There was no $V$ y initially when ball was kicked from roof....V yi $=0$

## Connections and Careers:

## Many athletes must have an understanding of projectile motion:

Ex: a hockey player may not understand the algebraic nuances of projectile motion, but he must have an innate sense of how projectiles work if he is to shoot a puck into the upper corner of the net.

Ex: Any sport where an object is thrown, such as baseball, basketball, and football, involves the RANGE equation ( $\mathrm{d}_{\mathrm{x}}=\mathrm{v}_{\mathrm{x}} \mathrm{x} \mathrm{t}$ ). Even though players do not think about physics during a game in order to make their plays, after years of practice they have " calibrated eyeballs" that permit them to know just how to throw a ball.

ball kicked at an angle

HORIZONTAL LAUNCHING:
Text Reference: 3.3



## TYPE 1

-- a projectile that is launched horizontally and lands BELOW the point of projection



Remember, projectiles that are launched horizontally:

## the angle $\boldsymbol{\theta}=\mathbf{0}$ degrees with the Horizontal


initial velocity in Y direction is $0.0 \mathrm{~m} / \mathrm{s}\left(V_{1 y}=0 \mathrm{~m} / \mathrm{s}\right)$

Horizontal velocity is constant ( $V_{x}$ is constant)

Vertical acceleration ( $a_{y}$ ) is

- $9.8 \mathrm{~m} / \mathrm{s}^{2}$

Horizontal acceleration ( $a_{x}$ ) is $0 \mathrm{~m} / \mathrm{s}^{2}$

Time is same for both the $x$ and $y$ dimensions of the motion

## Practice exercise 1

As a plane flies horizontally at $65.0 \mathrm{~m} / \mathrm{s}$, it releases a package from a height of $1.20 \times 10^{3} \mathrm{~m}$. (June 26)

(i)

What is the horizontal distance the package travels after it is released?
ii)

What is the final velocity of the package?

## Practice exercise 2

A golf ball is hit horizontally from a knoll that is 12.7 m above the level course. It leaves the tee traveling at $27.8 \mathrm{~m} / \mathrm{s}$. When it lands, how far will it be from the base of the knoll? ( Range?)


## Practice exercise 3

In a laboratory activity, students launch a toy car horizontally off a table with a speed of $3.6 \mathrm{~m} / \mathrm{s}$ as shown. If a 0.25 m wide target is placed 1.0 m from the base of the table, determine whether the car will hit the target. (August 2007)


"Oh, and I suppose it was me who said 'what harm could it be to give the chickens a book on nuclear physics?'"

## Unit 1

## SOLVING PROJECTILE MOTION EXERCISES:

Launching from an angle and
Landing Back at Point of Projection

Text Reference:
3.3



Thanks to the innoyctive labs of teacher Herb Krenley, physics quickly become Westvale High's most popular course.
a projectile is launched AT AN ANGLE and lands AT the same level as the point of projection


The velocity of a projectile is shown at various points along its path. Notice that the vertical component changes while the horizontal component does not. Air resistance is neglected.

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Both projectiles have the same launching speed. The initial velocity vector has a greater vertical component than when the projection angle is less. This greater component results in a higher path.
The horizontal component is less, so the range is less.


Remember, :


Angle $\boldsymbol{\theta}>\mathbf{0}$ degrees with the Horizontal
It has an initial velocity in $Y$ direction $V_{y}$
the horizontal velocity is constant ( $V_{x}$ is constant)
At the top of the path the vertical component ( $V_{y}=0 \mathrm{~m} / \mathrm{s}$ ), so the velocity there is the same as the horizontal component o velocity at all other points.
Time is same for both the $x$ and $y$ dimensions of the motion

## - Determination of the Time of Flight



If it takers a projectile 4 seconds to rise to its peak, then it will talce a total of 8 seconds to moree through the air from start to finish.

## VECTOR COMPONENTS OF A PROJECTILE

A ball's velocity can be resolved into horizontal (Vx) and vertical components (Vy).


## Practice exercise 1

When juggling two balls, the first ball was thrown from the right hand to the left hand at an angle of $60^{\circ}$ to the horizontal with an initial velocity of $2.5 \mathrm{~m} / \mathrm{s}$.
A) Calculate the time the ball remains in the air. (Assume the ball is caught at the same height from which it was released.)
B) How far apart should the juggler's hands be to catch the first ball?

## Practice exercise 2

A football is kicked at an angle to the horizontal of $\theta=$ $37.0^{\circ}$ with a velocity of magnitude $20.0 \mathrm{~m} / \mathrm{s}$.
A. Calculate the maximum height of the football?
B. The time of travel before it hits the ground?


## Activity



- Extra Read Text P.84-89
- Handout on "Projectile Motion"
- Text Practice Problems: Pages 115-116-\#20-28


Launching From an Angle and
Landing Above Point of Projection

....a projectile that is launched AT AN ANGLE and lands ABOVE the point of projection


## Example 2

The great Projecto is launched at $50^{\circ}$ above the horizontal at $30 \mathrm{~m} / \mathrm{s}$. For is stunt a net is set up 3.0 m above is launch height.

A) What is the final velocity of the Great Projecto
just before he hits the net
B) What distance does he travel horizontally (Range)?

## Example 2

An object is projected from the top of a building at an angle of $28^{\circ}$, as shown in the diagram, at a velocity of $15 \mathrm{~m} / \mathrm{s}$. If the object hits the ground 32 m from the base of the building, how high is the building?

## Example 3

A hockey players shoots the puck at $33.0 \mathrm{~m} / \mathrm{s}$ at an angle of $35^{\circ}$ with the ice surface. If he is 6.3 m from the boards and the boards are 3.0 m high, will the spectators be protected?


## SOLVING PROJECTILE MOTION EXERCISES:

Launching From an Angle and
Landing Below Point of Projection

....the projectile is launched AT AN ANGLE and lands BELOW the point of projection

....for the purposes of this course, you will not be expected to find the launch angle of a projectile


## Example 1

A ball rolls off an incline, as shown, at a velocity of $22 \mathrm{~m} / \mathrm{s}$. How far from $B$ will the ball hit the floor?


## Example 2

A soccer ball is kicked at 18.2m/s at 54 degrees above the horizontal (level playing field). The ball goes over the fence and clears and embankment and lands on a sandy beach 3.4 m below the level of the field.
A) How fast will it be going when it hits the beach?
B) How long was the ball in the air?


## Example 2

A tennis player lobs a ball in a high arc as shown. A radar clocks the ball at $10.9 \mathrm{~m} / \mathrm{s}$ and hits the court 1.83s after being struck by the tennis racket. As shown in the picture, the player has a reach of 0.75 m . How tall is the player?


## Solving Problems Involving Projectile Motion

: Where does the apple land?
A child sits upright in a wagon which is moving to the right at constant speed as shown. The child extends her hand and throws an apple straight upward (from her own point of view), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (a) behind the wagon, (b) in the wagon, or (c) in front of the wagon?


Ground reference frame

## Solving Problems Involving Projectile Motion

## The wrong strategy.

A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.


Critical Thinking

A zoo keeper invents a canon to shoot bananas to a monkey who is too shy to come down from the trees and eat.

If the monkey does not move, should the zoo keeper aim at, above, or below the monkey?


but in real life there is gravity so the zoo keeper should aim above the monkey!

If the monkey lets go of the branch at the instant the zoo keeper shoots the banana, should the zoo keeper aim at, above, or below the monkey to get the monkey the banana in mid-air?



He should aim at the monkey... anyone know why?


The Answer:

Because of gravity, the banana (like any projectile) will fall below this direct straight line. How far below? As far as the monkey falls in the same amount of time. So the monkey and the banana will fall the same vertical distance in the same time, and mid-air contact is made.


But doesn't it matter how hard he shoots the banana?

Nope, they'll still meet in mid-air just at different vertical positions

## high- <br> speed <br> shot

## Activity



- Handout on "Projectile Motion"-part 2


## STSE

- The physics of Juggling



## GOLF <br> DEMONSTRATION



## Core Laboratory \#1

- Initial Velocity of a projectile


